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'tHooft's Renormalon: Experimental and Number Theoretical Arguments for Physical Existence

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Abstract **– 'tHooft's renormalon, similar to instanton, is regarded by G. 'tHooft as a genuine singularity in the corresponding mathematical theory. However unlike instanton there is no mathematical proof for the existence of renormalon, nor is it taken to be definite real physics as distinct from a valuable mathematical object produced by the mathematical formulation of the theory. By contrast the present work will argue for the physical reality of the renormalon and proposes that this reality is strongly anchored in experimental evidence as well as number theoretical reasoning connected to Hardy's quantum entanglement as well as the theories of cosmic dark matter and dark energy. The final conclusion is that renormalons, as discovered thanks to the pioneering work in quantum field theory by D. Gross, A. Neven, B. Lautrup and particularly the enduring efforts of G. 'tHooft are not mathematical artefacts connected to summing diverging series of a perturbative solution, but rather a profound physics at the very deep roots of pre-quantum state where the traditional demarcation between physics and mathematics disappears or at a minimum is blurred as is the case in E-infinity Cantorian spacetime theory.**

Keywords **– Hardy's quantum entanglement, Gross heterotic superstrings theory, 'tHooft's renormalon, Entangleon, Cantorian spacetime, E-infinity, Dark energy, Dark matter, Einstein maximal energy density, Special relativity, Zero set quantum particle, Empty set quantum wave.**

I. Introduction

Renormalon [1-12] was introduced into quantum field theory by the Dutch theoretical physicist G. 'tHooft [1-4] unlike 'tHooft's instant as far as the present author is aware, there is no mathematical proof for the existence of this singularity and even less evidence for its physical reality [8-12]. The present work takes the view that there are sufficient indirect experimental evidence matched by number theoretical reasoning which goes a long way towards arguing for the physical reality of the renormalon as an elementary particle, albeit an exotic one connected to the topological simplictisity of the vacuum as well as our quantum spacetime which supports the existence of Hardy's quantum entanglement [13-16] as well as the triality of ordinary energy, dark matter and pure dark energy of our cosmos [8, 9].

II. Experimental and Number Theoretical Arguments for the Reality of 'tHooft's Renormalon

1.1. Background Information

The fabric of our experimental-number theoretical neural net like informational space will require numerical facts connected to the mathematics of 'tHooft's dimensional renormalization method [1-4]. Furthermore we will need the number theoretical interpretation of Hardy's quantum entanglement [13-16] in addition to the experimental verification of this entanglement and its physical bearing on the cosmic measurement and observation results obtained from COBE, WMAP and L2 supernova of the accelerated cosmic expansion [10]. This is a vital point because it is the main observational link for the so called missing dark energy density of our universe [17-43]. To do all of that we need to go back to von Neumann-Connes dimensional function of Penrose fractal tiling universe which we use to model the holographic boundary of our universe [22-25]. This in turn led us to set the pre-quantum particle to be modelled by the bi-dimensional zero set given by zero and ϕ where $\phi = \left(\sqrt{5}-1\right)/2$ while the cobordism of the zero set is given minus one and ϕ^2 so that it is naturally an empty set modelling the pre-quantum wave [24-30]. Lifting both sets to Kaluza-Klein five dimensional spacetime we arrive at a five dimensional spacetime multiplicative zero set volume $5\phi^5$ for the particle and to an additive five dimensional empty set volume $5\phi^2$ [22-30]. The total volume is consequently the sum, i.e. $\phi^2 + 5\phi^2$ which represents a two dimensional world sheet leading to a total topological energy equal to [22-30]

$$\begin{aligned} E &= (\text{Topological volume})\ \frac{1}{2}mv^2 \\ &= \left(\phi^5 + 5\phi^2\right)\left(\frac{1}{2}\right)mv^2 \\ &= (2)\left(\frac{1}{2}\right)mv^2 = mv^2 \end{aligned} \qquad (1)$$

Replacing v by the maximal speed of light one finds Einstein's famous formula which represents the largest energy density in the universe $E = mc^2$ [31]. That way we have dissected this classical energy to two parts, the ordinary energy part of the zero set particle ($\phi^5/2$) and the dark energy part of the empty set quantum wave ($5\phi^2/2$) [22-31]. In fact using more sophisticated topology for spacetime $E = mc^2$ could be dissected into three parts, namely that of ordinary energy ($5\phi^2/2$) plus a dark matter energy density coupled to a pure dark energy density [17-28]. All this was shown some time ago to be in near to perfect agreement with cosmic measurements and observations [8, 10, 21, 33]. The delightful surprise was however an unexpected agreement with the theory of Hardy's quantum entanglement and its experimental verification [13-16]. This exact solution of L. Hardy showed beyond any doubt that the quantum probability of

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two entangled quantum particles is about 9 percent [19]. However careful number theoretical considerations lead to the firm result that Hardy's entanglement probability is exactly ϕ^5 [22-31]. In other words the ordinary energy density of our universe is half of this value, i.e. $\phi^5/2$ for each particle and this is an experimentally founded results of fundamental quantum mechanics coupled to a fundamental result measured in quantum cosmology [17-31]. It remains still to show the relation between this result and 'tHooft's dimensional regularization procedure where the renormalon appeared for the first time [1-7].

2.2 *Background Information about 'tHooft's Dimensional Renormalization*

The importance for a theory to be renormalizable is well known to all working on Yang-Mills theory and related problems connected to extension of the standard model of elementary particles. It was the young 'tHooft who had the honour of solving a tricky problem by extracting an answer from a divergence at a singularity [1-7] and thus opened the door to the experimental detection of W^+, W^-, Z^o as well as the Higgs [19]. In a nutshell he used a perturbation like method connected with Borel summation and got his answer right by avoiding a pole singularity giving a finite answer to an otherwise infinity non-answer [1-7]. To do that he had to set the dimension of Einstein spacetime not as it normally is, namely D = 4 but he used a slightly smaller value by subtracting the perturbation parameter ε so to speak from D = 4 to become D = 4 – ε then he could let ε go to zero later on [1-7]. Never the less and as recounted on various semi formal occasions by 'tHooft, he and his at the time thesis advisor M. Veltman frequently thought that this method may be hinting at the possibility that spacetime may have a non-integer fractal dimension and strictly not the smooth D = 4 spacetime of Einstein [31]. As much as the two great physicists tried, they have never been able to give a final definite answer to this problem as per 'tHooft's own words to the audiences of many talks as well as to the present author in private discussions. It was many years later that the present author was able to deduce the exact value of the "missing" dark energy of the universe as a ratio between the dimension of a Kaluza-Klein spacetime to that of a fractal Kaluza-Klein universe [29, 30]

$$\gamma(D) = 5/(5+\phi^3) = 5\phi^2/2 \cong 95.5\% \tag{2}$$

At about the same time or some what later, the author noticed that a similar move can be made of 'tHooft's D = 4 – ε when it is divided by Einstein's D = 4 leading to precisely the same result provided the perturbation parameter ε be set equal to twice the magnitude of Hardy's entanglement, i.e. $2\phi^5 = k = \phi^3(1-\phi^3) = 0.18033988$ and which is at the same time exactly equal to four times the energy density of the ordinary energy of the cosmos as we discussed earlier on in the introduction, namely

$$k = (4)(\gamma(0)) = (4)(\phi^5/2) = 0.18033989 \tag{3}$$

Now we are in a position to present our utterly simple analysis to show that 'tHooft's renormalon is real.

III. Analysis of Renormalon and Arguments for its Physical Reality

The intriguing equality of the ratio between the fractal 'tHooft space D = 4 – ε where $\varepsilon = \phi^3(1-\phi^3)$ and Einstein space D = 4 on the one side and the dimension of the classical Kaluza-Klein space D = 5 to the fractal Kaluza-Klein space $D = 5+\phi^3$ as given by the following remarkable equation [22-31]

$$\frac{4-k}{4} = \frac{5}{5+\phi^3} \tag{4}$$

is worth putting under our transfinite numerical microscope as done on various similar occasions in E-infinity theory. In this case we easily realize that k integrated transfinitely several times by up scaling via the inverse of the golden mean 1 + ϕ = 1/ϕ would lead to 5+ϕ^2 because

$$\begin{aligned}(k)(1/\phi)^7 &= \phi^3(1-\phi^3)(1/\phi)^7 \\ &= k(d_c^{(8)}) \\ &= 6.18033989(1/\phi)^{8-1} \\ &= 5+\phi^3\end{aligned} \tag{5}$$

The above says more than just that $5+\phi^3$ is a scaling or pseudo integration of the renormalon k because it has a profound interpretation as a "topological" mass. The first part 5 is the topological mass of dark energy while ϕ^3 is the topological mass of the ordinary energy although ϕ^3 is the Hausdorff dimension of the surface of the empty set ϕ^2 so that ϕ^3 is not only the inverse of $4+\phi^3$ Cantorian spacetime dimension but also the Hausdorff dimension of the next emptier set after ϕ^2 which models the pre-quantum wave [25-30]. Remembering now that Hardy's quantum entanglement ϕ^5 is an experimentally tested reality and that similarly $\phi^5/2$ is a fact of cosmic measurement of ordinary energy we see that four ordinary energy elementary particles as well as two entangleon particles form one renormalon particle

$$\begin{aligned}k &= (4)(\phi^5/2) = (2)(\phi^5) = \phi^3(1-\phi^3) \\ &= 0.18033989\end{aligned} \tag{6}$$

It is a trivially simple logic to maintain that a compound elementary particle made from experimentally verified quantities is with a probability bordering on certainty also a real physical quantity and consequently renormalon is not just a theoretical manoeuvre [1-7] or a mathematical singularity [1-7] but rather a reality and even if it looks like a mathematical singularity it is still a particle in the same sense that we may look at elementary particles as mini black hole singularities in our spacetime manifold.

IV. Discussion

E-infinity fractal-Cantorian spacetime met 'tHooft's ideas when it was realized that dimensional renormalization implies Cantorian, i.e. fractal spacetime [4-46]. The next step in this development was the present

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realization, namely that if spacetime is really a fractal, then all the physical consequences of this fractality is necessarily as real as its origin. The point seems obvious and all speaks for the fact that behind the mathematical Hausdorff fractal dimension of space is a real physical phenomenon. Let us list these mathematical quantities with profound test for physical reality:

(a) Quantum entanglement is as real as the quantum world can be and although the Hardy value ϕ^5 seems at first sight surreal to the extent that Hardy himself overlooked the fact that the probability which he found and rounded from 0.09017, to about 9 percent is in reality 0.0916994393 and is equal to the golden mean $\phi = \left(\sqrt{5}-1\right)/2$ to the power of the five dimensional Kaluza-Klein spacetime, i.e. ϕ^5 [30]. In fact even the great British mathematician Sr. R. Penrose in his monumental work [19] did not notice the involvement of the golden mean in Hardy's experimentally confirmed probability.

(b) All measurements and observations lead to the conclusion that dark matter and dark energy are real and that the percentages of energy density in the universe are 4.5% for ordinary measureable energy, 22% for dark matter energy and 73.5% for pure dark energy [33] so that for the dark sector we have 22 + 73.5 = 95.5 and for the ordinary sector we have 4.5. Our E-infinity theory made a very precise prediction which linked up immediately to Hardy's confirmed result, namely that the ordinary energy density of the cosmos is not simply 22% but rather (22 + k)% where $k = \phi^3(1-\phi^3)$ is the exact value which when used in place of 'tHooft's ε in his dimensional regularization method leads to the same dark energy density and consequently the same ordinary energy density because of the simple golden mean number theoretical relation involved in the following equation [22-33]

$$\frac{D-\varepsilon}{D} = \frac{4-k}{4} = 5\phi^2/2 = 1-(\phi^5/2) = \frac{4-2\phi^5}{4} = \frac{5}{5+\phi^3} \quad (7)$$

Thus all these quantities were the result of accurate measurement for which at least two Nobel Prizes in Physics and Cosmology were awarded [21]. It follows then that the $\varepsilon = k = 2\phi^5$ used in the above is real, i.e. 'tHooft renormalon as characterized by the dimensional index k is also physically real [8-12].

It may be that the reason for the delayed recognition that 'tHooft renormalon is not only a true singularity in the fabric of spacetime [1-4] but also a real quasi elementary particle can be found in its connection to the golden mean making many scientist at least slightly shy to dwell on the aspect which more often than not is considered to be the playground of esoteric and mystical thoughts. In a sense the golden mean with all its ramifications in hard core science is the victim of its phenomenal success. However the truth is that there is no mystique and fog about this number bt there is a great deal of hard core number theory in it which means there is a great deal of physics in it hiding as number theory and visa versa because at a truly deep level of science, the division between what is physics and what is logic and mathematics is largely blurred and in fact misguided [28]. The new science of quantum chaos for instance as well as nonlinear dynamics and Feigenbaum constants would not have seen the light without the golden mean based KAM theorem and the golden mean renormalization groups [22-25] to mention only some obvious examples [44]. From the preceding discussion we are firmly of the opinion that 'tHooft's renormalon is real physics and it is only a matter of time until we can give clear cut experimental evidence for the reality of 'tHooft's renormalon.

V. The Three Fundamental Types of Quantum, Cosmological Pre-Mass

To make the present paper reasonably self contained, we feel we should give a reasonably detailed account of our E-infinity conception of the mathematics of topological pre-mass involved in the present analysis and which leads to consistent results on both the theoretical and the observational and measurement levels [8-46]. It turns out that there are three fundamental distinct types of mass. These are (a) the Hausdorff mass, (b) the topological mass and (c) the combined or mixed topological-Hausdorff mass. We discuss each one separately first.

Remembering that in a one dimensional random Cantor set the maximal "topological" speed is the Hausdorff dimension ϕ and that it can be equated to a topological speed of light, then we can write the kinetic energy of Newton $E = (1/2)mv^2$ in different notation as $E = (1/2)\phi^2\phi^3 = \phi^5/2$ which means the ordinary energy density is

$$\gamma(0) = \phi^5/2 \quad (8)$$

because ϕ^3 is the Hausdorff mass of ordinary energy [33]. In a similar way we find the dark energy density to be

$$E = \frac{1}{2}(5)(\phi^2) = \gamma(D) = 1-(\phi^5/2) \quad (9)$$

On the other hand the mixed mass of pure dark matter energy and pure dark energy density becomes [33]

$$E = \frac{1}{2}(3\phi^2)(\phi^2) + \Delta = \gamma(DM) = (1.5)(\phi^4) + \Delta \quad (10)$$

and

$$E = \frac{1}{2}(4-\phi^4)(\phi^2) - \Delta = \gamma(PD) = \left(\frac{7+\phi^3}{10}\right) - \Delta \quad (11)$$

respectively and Δ is the cosmological coupling stemming from the holographic boundary of the universe. Thus Δ, which cancelled out for the dark sector as a whole, is equal to the inverse of the compactified Kleinian group dimension describing the symmetry of the Penrose-Connes universe [33].

$$\Delta = \frac{1}{SL(2,7)+16k} = \frac{1}{336+(16)\phi^3(1-\phi^3)} = 0.002950849718 \quad (12)$$

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To gain a somewhat intuitive feel for the meaning of the subtle but definite distinction between three types of mathematical quantum-like mass corresponding to four types of energy densities, let us simplify each expression to the bone and disregard for a moment the essential high numerical accuracy of our calculations. Thus let us drop Δ which cancels out any case in the sum. Starting in reverse order for pure energy we see that [33]

$$\gamma(PD) \simeq \left(\frac{1}{2}\right)(4)(\phi^2) \tag{13}$$

and consequently it depends on 4 dimensional objects as a "mathematical" mass which may be seen also from

$$\gamma(PD) \simeq 2\phi^2 \tag{14}$$

as a two dimensional membrane acting on the empty set or quantum wave vacuum given by ϕ^2. Alternatively it is the energy of an almost four dimensional object. Next for pure dark matter we have [33]

$$\gamma(DM) \simeq \frac{1}{2}(1.14589)(\phi^2) \simeq \frac{1}{2}(1)(\phi^2) \tag{15}$$

which looks like a one dimensional fractal string (1.14589) acting on the vacuum, i.e. ϕ^2.

The case of dark energy, lumping pure dark energy and pure dark matter together is by contrast quite clear cut. The mass is a Kaluza-Klein five dimensional object acting on the vacuum ϕ^2 [33, 40, 45].

Finally ordinary energy is the very exception being a straight forward fractal object with a Hausdorff mass given exactly by the inverse of the Hausdorff dimension of the core of fractal spacetime $4+\phi^2$, i.e. $1/(4+\phi^3)=\phi^3$. It is obvious that it is a zero dimensional fractal connected to the zero set pre-quantum particle (0, ϕ) as is well known from von Neumann-Connes' dimensional function of Penrose noncommutative fractal space. Consequently ordinary energy stands out as connected to the quantum particle and only to the quantum particle and that is why in all tests of E = mc^2 which are conducted in 3 + 1 classical space and time we could not ever detect directly the triality of E = mc^2 and its three main components [35]. To discover the absence of dark matter and dark energy on cosmic scales we needed to find out indirectly that something is not as simple as we thought and that we have two main sources of energy, namely one connected to the zero set quantum particles which account for 4.5% of the energy density and the other is connected to the empty set quantum wave and accounts for the 95.5% density which we cannot measure directly and is called dark energy. In turn this dark energy may be divided into two parts where the dark matter is connected to fractal strings and accounts for 22% of the total energy density while pure dark energy is connect to fractal-like 4 dimensional objects and accounts to almost 73.5% of the total energy density of the cosmos [33].

To sum up, based on the above, our universe can be conceived to be a five dimensional space at the core [29-35]. It is almost smooth but not completely due to an additional fractal Hausdorff component (ϕ^3) so that our universe is really $5+\phi^3$ at the core. This is something resembling a fractal "self similar" Kaluza-Klein universe enveloping a fractal Einstein space. This little ϕ^3 is however enormously important because it is the source of all ordinary energy and ordinary matter in the universe. The 5D bulk on the other hand may be divided into almost one dimensional fractal-like strings constituting dark matter plus almost four dimensional objects which are the source of pure dark energy. Together the strings and the four dimensional objects form the solid 5D core of the classical Kaluza-Klein universe. However this is only when taken together. Looking more carefully, the part of which the 5D is made is not entirely smooth but rather rugged and definitely related to the fractal character of ordinary energy with its ϕ^3 Hausdorff mass. That is the best we can do at the moment to explain this weird picture of reality without using the mathematical language of transfinite sets. However one should admire the neatness of the following equation which came naturally without forcing anything in our analysis, namely [22-33]

Pure dark energy mass + dark matter mass
= $(D-\phi^4)+(3\phi^2)$
= 3.854101966+1.145292035 = 5 (16)
= The dimension of the classical Kaluza-Klein theory
= mc^2 of Einstein using the fractal Kaluza-Klein theory

Again this leads us to a new profound way of writing E = mc^2 of Einstein using the fractal Kaluza-Klein theory, namely [22-33]

$$E = \frac{\left\{\phi^3\left[\left(1+\phi^4\right)+\Delta\right]+\left[\left(4-\phi^4\right)-\Delta\right]\right\}}{5+\phi^3}mc^2$$

= (ordinary energy γ + dark matter γ + pure dark energy γ) mc^2 (17)

where $\Delta=(5+\phi^3)/(336+16k)$.

VI. General Conclusions

The great service which nonlinear dynamics, chaos and in particular fractals rendered to theoretical physics is that it brought transfinite set theory and some of the most important results of pure mathematics much nearer to theoretical physics and quantum cosmology. The development is complimentary to some of the pioneering work by E. Witten and C. Fafa who realized that superstring theory, powerful as undoubtedly it is, would not be sufficient unless complimented by a topological superstring theory.

In the present work we have de facto shown that the superstrings program can gain a great deal by incorporating the modern mathematical machinery of measure theory and fractal topology starting from the basic building blocks, namely the zero set and the empty set [28-30] as well as the ingenious notion of the degree of emptiness of an empty set introduced first by B.

Mandelbrot and extended by the present author [49-50]. That way we keep a handle on the supposedly unruly landscape of string vacuum and find direct solutions to traditionally very complex problems by a minimal transfinite modification of basic topological invariants, for instance Einstein's D = 4 becomes $D=4+\phi^3$ of E-infinity or $D=4-k$ of 'tHooft-Veltman fractal spacetime [22-30]. Even more surprising is the fractal version of D = 11 Witten's M-theory which transforms to $D=11+\phi^5$ [30] or Kaluza-Klein D = 5 transmuting to the incredible $D=5+\phi^5$. Far from being only mathematical manoeuvres, we were able to use these new tools for all the preceding innovations to find exact solutions to the ordinary and dark energy sector of cosmic energy which are in astounding agreement with accurate cosmic measurements and observations. We may be a little too optimistic and enthusiastic about the future development of theoretical physics but we really think that the inclusion of nonlinear dynamics and chaotic fractals on a fundamental level in high energy physics and quantum cosmology is heralding a new era of progress unparalleled in the relatively short history of modern science. The conclusion of the present work that 'tHooft's renormalon is physically real and can in principle be found experimentally in the laboratory is an example of what theoretical investigation can do in the future and how anything logical and mathematical must be at the end of the day, physical and real [51-53].

REFERENCES

[1] G. 'tHooft: Dimensional regularization and renormalization group. Nuclear Physics, B61, 1973, pp. 455-468.

[2] G. 'tHooft: Dimensional regularization (unpublished). Marseille, France Conference, 19-23 June, 1972.

[3] G. 'tHooft: Computation of the quantum effects due to a four dimensional pseudo particle. In 'Instantons in Gauge Theory'. Editor: M. Shifman, World Scientific, Singapore, 1994, pp. 70-80.

[4] M.S. El Naschie: On 'tHooft dimensional regularization in E-infinity space. Chaos, Solitons & Fractals, 12(5), 2001, pp. 851-858.

[5] M.S. El Naschie: On a general theory for quantum gravity. In 'Science of the Interface. Editor H. Diebner, T. Druckrey and P. Weibel. Genista Verlag, Tübingen, Germany, 2001, pp. 875-880.

[6] G. 'tHooft: Can we make sense out of 'Quantum chromodynamics'. In 'The Whys of Subuclear Physics'. Editor: A. Zichichi, Proceedings of the Int. School, Evice, Italy. Springer-Plenum Press, New York, USA. 1977.

[7] D.J. Gross and A. Neven: Dynamical symmetry breaking in asymptotically free field theories. Physical Review, D10(10), 1974, pp. 3235-3253.

[8] Mohamed S El Naschie: From a dual Einstein-Kaluza spacetime to 'tHooft renormalon and the reality of accelerated cosmic expansion. Journal of Modern Physics, 8(8), 2017, pp. 1319-1329.

[9] Mohamed S. El Naschie: Dark energy and its cosmic density from Einstein's relativity and gauge fields renormalization leading to the possibility of a new 'tHooft quasi particle. The Open Journal of Astronomy. 8, 2015, pp. 1-17.

[10] Mohamed S El Naschie: 'tHooft renormalon and the reality of accelerated cosmic expansion. SciFed Journal of Quantum Physics, 1(1), 2017, pp. 1-7.

[11] M.S. El Naschie: Cosmic dark energy from 'thooft's dimesnionl regularization and Witten's topological quantum field pure gravity. Journal of Quantum Information Science, vol. 4(2), 2014 , pp. 83-91.

[12] Mohamed S. El Naschie: On a new elementary particle from the disintegration of the symplectic 'tHooft-Veltman-Wilson fractal spacetime. World Journal of Nuclear Science and Technology, 4(4), 2014, pp. 216-221.

[13] M.S. El Naschie: Quantum entanglement as a consequence of a Cantorian micro spacetime geometry. Journal of Quantum Information Science, 1, 2011, pp. 50-53.

[14] Mohamed S. El Naschie: A resolution of cosmic dark energy via quantum entanglement relativity theory. Journal of Quantum Information Science, 2013, 3(1), pp. 23-26.

[15] M.S. El Naschie: The quantum gravity Immirzi parameter – A general physical and topological interpretation. Gravitation and Cosmology, Vol. 19(3), 2013, pp. 151-155.

[16] Mohamed S. El Naschie: Electromagnetic —Pure Gravity Connection via Hardy's Quantum Entanglement. Journal of Electromagnetic Analysis and Applications, 6(9), 2014, pp. 233-237.

[17] L. Amendola and S. Tsujikawa: Dark Energy: Theory and Observation, Cambridge University Press, Cambridge, UK, 2010.

[18] S. Weinberg: Cosmology. Oxford University Press, Oxford, UK. 2008.

[19] R. Penrose: The Road to Reality. Jonathan Cape, London. (2004).

[20] P. Ruiz-Lapuente, Dark Energy, Observational and Theoretical Approaches. Cambridge University Press, Cambridge, 2010.

[21] R. Panek: The 4% Universe. One World Publications, Oxford, UK, 2012.

[22] L. Marek-Crnjac, Ji-Huan He: An invitation to El Naschie's theory of Cantorian space-time and dark energy. International Journal of Astronomy and Astrophysics, Vol. 3(4), 2013, pp. 464-471.

[23] Leila Marek-Crnjac: On El Naschie's Fractal-Cantorian Space-Time and Dark Energy—A Tutorial Review. Natural Science, 7(13), 2015, pp. 581-598.

[24] M. Helal, L. Marek-Crnjac and Ji-Huan He: The three page guide to the most important results of M.S. El Naschie's research in E-infinity quantum physics. Open Journal of Microphysics, 2013, 3, pp. 141-145.

[25] M.S. El Naschie: A unified Newtonian-relativistic quantum resolution of supposedly missing dark energy of the cosmos and the constancy of the speed of light. International Journal of Modern Nonlinear Theory & Application, 2013, 2, pp. 43-54.

[26] Mohamed S. El Naschie: Compactified dimensions as produced by quantum entanglement, the four dimensionality of Einstein's smooth spacetime and 'tHooft's 4- ε fractal spacetime. American Journal of Astronomy & Astrophysics, 2(3), 2014, pp. 34-37.

[27] Mohamed S. El Naschie: Dark Energy from Kaluza-Klein Spacetime and Noether's Theorem via Lagrangian Multiplier Method. Journal of Modern Physics, 4(6), 2013, pp. 757-760.

[28] Mohamed S. El Naschie: Einstein's dark energy via similarity equivalence, 'tHooft dimensional regularization and Lie symmetry groups. International Journal of Astronomy & Astrophysics, 6, 2016, pp. 56-81.

[29] Mohamed S. El Naschie: Einstein-Kaluza combined spacetime as the optimal and simplest framework to compute and understand dark matter, pure dark energy and measurable ordinary energy. Natural Science, 9(8), 2017, pp. 241-244. Letter to the Editor.

[30] Mohamed S. El Naschie: On a fractal version of Witten's M-theory. Journal of Astronomy & Astrophysics, 6(2), 2016, pp. 135-144.

[31] A.J. Babchin and M.S. El Naschie: On the real Einstein beauty E = kmc^2. World Journal of Condensed Matter Physics, 6(1), 2016, pp. 1-6.

[32] Mohamed S. El Naschie: Quantum dark energy from the hyperbolic transfinite Cantorian geometry of the cosmos. Natural Science, 8, 2016, pp. 152-159.

[33] Mohamed S. El Naschie: A combined Heterotic string and Kähler manifold elucidation of ordinary energy, dark matter, Olbers's paradox and pure dark energy density of the cosmos. Journal of Modern Physics, 8(7), 2017, pp. 1101-1118.

[34] Mohamed S. El Naschie: Completing Einstein's spacetime. Jour-

-nal of Modern Physics, 7(15), 2016, pp. 1972-1994.
[35] M.S. El Naschie: From E = mc2 to E = mc2/22 – A short account of the most famous equation in physics and its hidden quantum entangled origin. Journal of Quantum Information Science, 4, 2014, pp. 284-291.
[36] Hans Hermann Otto: Should we pay more attention to the relationship between the golden mean and Archimedes' constant. Nonlinear Science Letters A, 8(4), 2017, pp. 410-412.
[37] S.A. Olsen: Riding the numbers ratios and resonant states of consciousness. Sociology & Anthropology, 5(1), 2017, pp. 72-75.
[38] Mohamed S. El Naschie: Application of Dvoretzky's theorem of measure concentration in physics and cosmology. Open Journal of Microphysics, 5, 2015, pp. 11-15.
[39] Mohamed S. El Naschie: Topological-geometrical and physical interpretation of the dark energy of the cosmos as a 'halo' energy of the Schrodinger quantum wave. Journal of Modern Physics, 2013, 4, pp. 591-596.
[40] Mohamed S. El Naschie: Why E is Not Equal mc^2. Journal of Modern Physics, 5(9), 2014, pp. 743-750.
[41] Mohamed S. El Naschie: The measure concentration of convex geometry in a quasi Banach spacetime behind the supposedly missing dark energy of the cosmos. American Journal of Astronomy & Astrophysics, 2014, 2(6), pp. 72-77.
[42] Mohamed S. El Naschie: The hidden quantum entanglement roots of $E = mc^2$ and its genesis to $E = mc^2/22$ plus $mc^2(21/22)$ confirming Einstein's mass-energy formula. American Journal of Electromagnetics and Applications, 2014, 2(5), pp. 39-44.
[43] Mohamed S. El Naschie: Banach spacetime-like Dvoretzky volume concentration as cosmic holographic dark energy. International Journal of High Energy Physics, 2(1), 2015, pp. 13-21.
[44] M. S. El Naschie: A review of E-infinity and the mass spectrum of high energy particle physics. Chaos, Solitons & Fractals, Vol. 19, No. 1, 2004, pp. 209-236.
[45] A. Einstein: Ether and the theory of relativity. 5th May 1920 at the University of Leiden, Holland. Published by Methuen & Co. Ltd., London, UK, 1922. www.history.mcs.andrews.ac.uk/Extras/Einstein_ether.html
[46] L. Marek-Crnjac: Cantorian Space-Time Theory: The Physics of Empty Sets in Connection With Quantum Entanglement and Dark Energy. Lambert Academic Publishing, Saarbrucken, Germany. ISBN: 978-3-659-12876-9, 2013.
[47] C. Fafa: Superstrings and topological strings at large N. Journal of Mathematical Physics, 42(7), 2001, pp. 2798-2817.
[48] Mohamed S. El Naschie: The Aether as the source of the cosmic dark energy sector. International Journal of Innovation in Science and Mathematics, 5(6), 2017, pp. 177-181.
[49] Mohamed El Naschie and Ji-Huan He: Tesla's dream from a modern quantum spacetime view point. Nonlinear Science Letters A, 9(1), 2018, pp. 36-43.
[50] Mohamed S. El Naschie: The Aether of spacetime physics is the empty set of pure mathematics. Natural Science, 9(9), 2017, pp. 289-292.
51] Mohamed S. El Naschie: Spacetime as a new frontier advanced material with applications in physics, engineering, chemistry and cosmology. Advanced Material Physics and Chemistry, 7(9), 2017, pp. 347-352.
[52] Mohamed S. El Naschie: The physics, mathematics and common sense of cosmic dark energy and spacetime extra dimensions. International Journal of Innovations in Science and Mathematics, 5(6), 2017, pp. 201-204.
[53] Mohamed El Naschie: Elements of a new set theory based quantum mechanics with applications in high energy quantum physics and cosmology. International Journal of High Energy Physics, 4(6), 2017, pp. 65-74.

* This paper is dedicated to Prof. Gerardus 'tHooft.

AUTHOR'S PROFILE

Professor **M.S. El Naschie** was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl-Ing) diploma, equivalent to a Master's degree and Chartered Structural Engineer. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK, Cairo University, Egypt and is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt.

Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work is cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor.

Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 74 and his i-10 index is 753 according to Google Scholar Citation.