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Hemodynamics: Macroscopic and Microscopic Modeling

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***Abstract* – This article presents a work of modeling blood circulation. More specifically, we are interested in macro circulation and microcirculation. These two circulations depend on the size of the blood vessels.**

***Keywords* – Mathematical Modeling, Macroscopic Scale, Microscopic Scale, Flow, Diffusion, Convection, Reynolds Number, Viscosity, Hemodynamics, Blood Vessels, Microscopic Circulatory System, Macroscopic Circulatory System.**

I. Macroscopic Model

We study a first model of a Newtonian, viscous, permanent fluid flow by taking into consideration the characteristics of the walls (vibration, elasticity, pressure) on the blood flow in a cavity.

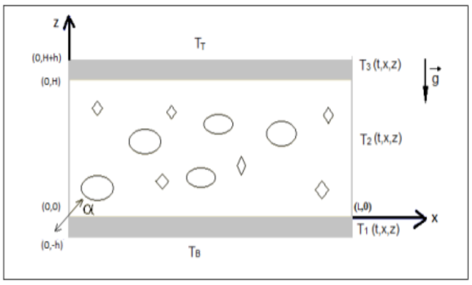

Fig. 1. Macroscopic flow.

The considered model is illustrated in Figure 1. It consists of a cavity placed between two thin plates, of the same thickness and the same thermal conductivity. Also, the conductivity of the plates is usually different from the conductivity of the cavity. On its outer surfaces $z = -h$ and$z = H + h$, we impose fixed temperatures at the edges. All the walls are rigid and the vertical walls: $x = 0$ and $x = L$, are adiabatic.

The cavity is saturated by a Newtonian and incompressible fluid. The effective conductivity of this set is different from those of the plates. This model was worked with a cavity containing a fluid flowing in a porous medium in the book [6].

For the modeling, we use the conservation laws of the quantity of movement, continuity, conservation of species and conservation of energy [7]. The functions are the flow velocity $\vec{V} = v(t,x,z)$, the pressure $P = P(t,x,z)$, the density $C = C(t,x,z)$ and the temperature$T = T(t,x,z)$.

$$\begin{cases} \nabla.\vec{V} = 0 \\ \frac{\partial \vec{V}}{\partial t} + \overrightarrow{(\vec{V}.\nabla)}.\vec{V} + \nabla P = 0 \\ \frac{\partial T}{\partial t} + \vec{V}.\nabla T + \nabla^2 T = 0 \\ \frac{\partial C}{\partial t} + \vec{V}.\nabla C + C.div\vec{V} = 0 \end{cases}$$

II. MICROSCOPIC MODEL

Going from the macroscopic scale to the microscopic scale, the physical laws taken into consideration change. Indeed, by changing scale, several phenomena seem to be quasi-stationary, or even non-existent as this is the case of convection. In the first paragraph, this phenomenon has a great importance in the transference. In this paragraph, we are interested in the study of the movement of small particles in flows of viscous fluids, in the vicinity of a plane wall.

This model admits several applications in the study of the various flows used in micro fluidics [2]. More explicitly, this work concerns the theoretical study of the behavior of a solid spherical particle on which a slip condition applies. This particle is injected into a shear flow of a viscous, Newtonian and incompressible fluid which flows in the vicinity of a flat and solid wall on which a slip condition is also exerted, as explained in figure 2.

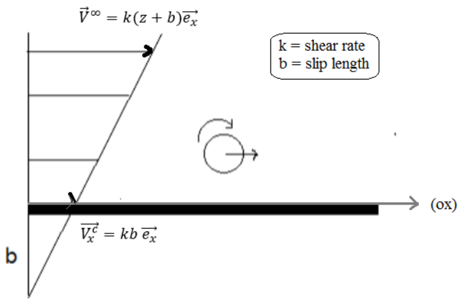

Fig. 2. Microscopic flow [5].

This flow can be calculated analytically, as in work [1] and [5]. The flow problem is modeled by the Stokes equations around a particle and parallel to a wall which enjoy slip conditions: $\begin{cases} \nabla.\overrightarrow{V^c} & = & 0 \\ \nabla P^c & = & \nabla^2.\overrightarrow{V^c} \end{cases}$

As these equations are linear [8], the problem can be decomposed in short, problems concerning elementary movements of the sphere: translation and rotation of a particle in a fluid primarily at rest, then in motion parallel to the wall for a fixed particle. A study on the boundary conditions on the sphere, the wall and at infinity was presented in chapter 4, of the book [6].

III. APPLICATION: HEMODYNAMICS

The mechanics of blood flow is called Hemodynamics. In other words, hemodynamics is the part of biophysics that studies the laws of blood flow in the vessels. These flow laws depend on flow rate, pressure, velocity and other parameters. Blood is a real fluid, that is to say, it undergoes internal friction during its flow through the blood vessels. Internal friction comes from two sources. The first is afriction between the molecules that make up the fluid. The second is friction between the fluid and the walls of the vessels. This internal friction, called viscosity, can lead to the release of heat. This friction is called viscosity. Blood is a viscous fluid. It is said to be real.

Fluid statics is characterized by pressure, while fluid dynamics is characterized by flow. As a liquid, blood is subject to the laws of both disciplines. On the one hand, it obeys the laws of hydrostatics, due to the differences in height between the head, the heart and the lower limbs and on the other hand, it responds to the laws of hydr-

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-odynamics, due to the heart-pulsating flow.

Hemodynamics takes into account two parameters of blood flow. On the one hand, blood contains macromolecules with specific properties and deformable cells. On the other hand, blood vessels are deformable and their walls have their own tension, elasticity and inertia.

3.1. *The Anatomy of the Blood Circulatory System*

The blood circulatory system is made up of the heart and blood vessels. The anatomy of the blood circulatory system is the study of the structure and shape of the heart and blood vessels, and the relationships between them. The heart plays the role of a pump that provides the blood with enough energy to circulate in the human body and reach all the organs. Blood vessels are classified according to their dimensions and their function in carrying blood towards the heart or in the opposite direction.

In this work, we are interested in the vascular system made up of arteries, arterioles, veins, venules and capillaries, as explained in Table 1. These efferent ducts branch out, increasing in number and decreasing in caliber as they move away from the heart. Blood vessels with a diameter greater than 0.1 mm are part of the macro circulation. Vessels visible only under a microscope are part of the microcirculation. Blood is viscous liquid. The viscosity of blood depends on the hematocrit, the deformability of the red blood cells and the temperature. Blood is a non-Newtonian fluid. However, to apply the above models, we consider it as a suspension of cells in plasma. Plasma is a Newtonian liquid. Its viscosity is $1.6\ 10^{2} Plat\ 37^{\circ}C$.

Table 1. Characteristics of flows in blood vessels.

Vessel	**Diameter (m)**	**Velocity** $(m.s^{-1})$	**Re**	**Flow**
Aorta	2.5×10^{-2}	0.4	2.8×10^{3}	Transient
Artery	4×10^{-2}	0.1 -0.4	1.1×10^{3}	
Vein	5×10^{-3}	0.003 – 0.05	4.1	Laminar
Capillary	8×10^{-6}	0<0.001	<0.002	
Venule	2×10^{-5}	<0.003	<0.017	
Arteriole	3×10^{-5}	0.001 – 0.1	0.08 -0.8	

When the plasma flow is laminar, the velocity distribution is parabolic in a straight section and Poiseuille's law is applicable.

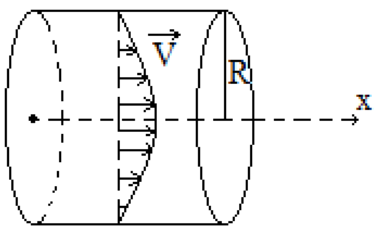

Fig. 3. Poiseuille's law applied in 3D, laminar flow.

Plasma flow is never turbulent except in cases of illness or fever. But it is transient in aorta and arteries due to high pressure system. When the value of the flow velocity gradient increases, red blood cells accumulate around the axis of the vessel; under these conditions, the velocity profile is flattened. The hematocrit is high but surrounded by a purely non-viscous plasma peripheral. This concentration around the axis of the blood vessel allows it to be studied in 2D.

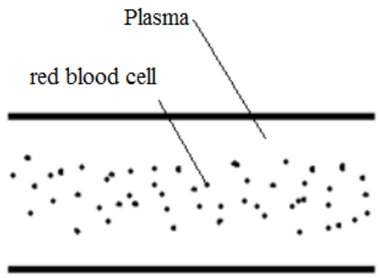

Fig. 3. Axis circulation.

The velocity profile according to Poiseuille's law becomes:

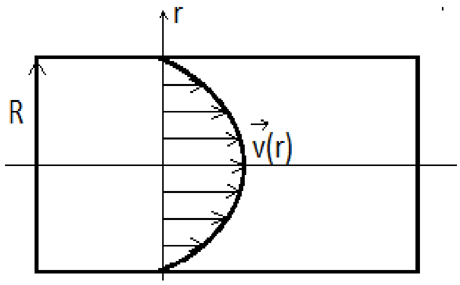

Fig. 1. Poiseuille's law applied in 2D, laminar flow.

For more details on hemodynamics, you can consult the book [6].

3.2. *Macroscopic Circulation Modeling*

Blood vessels that are part of the macroscopic circulatory system are classified according to the direction of blood flow towards the heart or the opposite direction. The arterial system essentially contains the arteries and arterioles. It allows blood to flow from the heart to the organs. The venous system contains veins and venules. It brings blood back to the heart to be transferred to the lungs for gas-blood exchange. Blood vessels have an elastic and expandable character. For example, an artery can be considered as a Windkessel damper[10]. Another example is the vasomotion of the arterioles. However, in this work, we assume that they are rigid.

At rest, the blood can be considered Newtonian and the flow is laminar. An example taken from [10] explains that at the level of the aorta, the Reynolds number is of the order of 1500, if we consider the following data: the average flow velocity is $25\ to\ 30\ m^{-1}$, the diameter of the vessel is $2\ cm$, the density is $10^3 kg.m^{-3}$ and the blood viscosity is $4.10^{-3} Pl$.

In [4], the authors model the blood flow by the Navier-Stokes equation using the two expressions of the continuity equation and the motion equation.

In this modeling work, we use the laws of conservation of momentum, continuity, conservation of species and conservation of energy.

The equations governing the macroscopic model are:

3.3. *Microscopic Circulation Modeling*

The blood vessels that are part of the microscopic circulatory system are the blood capillaries. Their basic structures are only perceptible under light microscopy or with electron microscopes. They are classified as continuous, fenestrated or discontinuous capillaries. In addition, they form a capillary network.

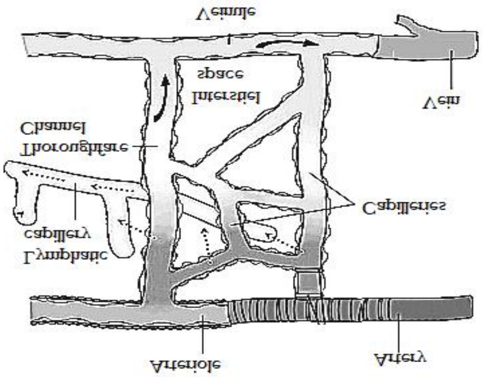

Fig. 5. Capillary bed [3].

The diameter of the systemic capillaries is $2\ to\ 10\mu m$ is of the same order as red blood cells $7\ to\ 8\ \mu m$. In this case, the blood flow is only carried out by the deformation of the red blood cells. Capillaries can be venous or arterial. They ensure the exchange of nutrients, oxygen and carbon dioxide between plasma and interstitial fluids by natural diffusion.

As a result, we can restrict ourselves to modeling moving cells to a spherical particle moving close to the capillary wall. Thus, the work done in [1] and [5] can be applied.

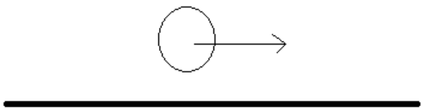

Fig. 6. A spherical particle in translation [1].

The equations governing the model of displacement of a particle in translation close to a wall are described in [1] by: $\begin{cases} \nabla.\overrightarrow{V^c} & = & 0 \\ \nabla P^c & = & \nabla^2.\overrightarrow{V^c} \end{cases}$.

IV. Conclusion

Partial differential equations can illustrate many phenomena as stated in [9]. Hemodynamics is a biophysical model that illustrates the importance of mathematical modeling scale. Some phenomena such as convection are highlighted for the macroscopic model, others are for the microscopic model such as shear.

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