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Integral Points On The Non - Homogeneous Cubic Equation With Five Unknowns $x^3 - y^3 = z^3 - w^3 + 12t^2$

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***Abstract*—The cubic Diophantine equation with five unknowns represented by $x^3 - y^3 = z^3 - w^3 + 12t^2$ is analyzed for finding its non-zero distinct integral solutions. Different patterns of solutions for the equation under consideration are presented. The relations between the integral solutions and special numbers namely Polygonal numbers, Pyramidal number, pronic number, Centered Pyramidal numbers are exhibited**

***Key Words*— Homogeneous cubic, Cubic with five unknowns, Integral solutions, integral points, Special numbers.**

I. Introduction

The cubic equation offers an unlimited field for research because of their variety [1],[16]. In particular one may refer [2]-[11] for the special cases of cubic diophantine equations with four unknowns. In [12]-[15], the cubic equations with five unknowns is studied for its non-zero integral solutions. This communication concerns with another interesting cubic equation with five unknowns given by $x^3 - y^3 = z^3 - w^3 + 12t^2$ for determining its integral solutions. A few interesting relations between the solutions are presented.

Notations

- $t_{m,n}$ - Polygonal number of rank n with size m.
- P_n^m - Pyramidal number of rank n with size m.
- CP_n^m - Centered pyramidal number of rank n with size m.
- $P(n)$ - Pronic number of rank n.
- $f_{m,s}^n$ - m-dimensional figurate number of rank n with s sides.

II. Method Of Analysis

The cubic diophantine equation with five unknowns to be solved for getting non-zero integral solution is

$$x^3 - y^3 = z^3 - w^3 + 12t^2 \quad (1)$$

It is observed that (1) is satisfied by the following quintuples:

$$(x, y, z, w, t) = (2r^2 + s^2 + 1, 2r^2 + s^2 - 1, 2r^2 - s^2 + 1, 2r^2 - s^2 - 1, 2rs),$$

$(2k^2 + 2, 2k^2, 2k^2, 2k^2 - 2, 2k)$ and $(3k + 1, 3k - 1, k + 1, k - 1, 2k)$.

In addition to the above solutions, we present below the different patterns of solutions to (1).

A. Pattern I

Introducing the linear transformations

$$x = c + 1\,;\, y = c - 1\,;\, z = a + 1\,;\, w = a - 1 \quad (2)$$

in (1), it is written as

$$a^2 + 2t^2 = c^2 \quad (3)$$

Rewrite (3) as

$$a^2 + 2t^2 = c^2 * 1 \quad (4)$$

Assume $c = u^2 + 2v^2$ (5)

Write 1 as $1 = \frac{(1+i2\sqrt{2})(1-i2\sqrt{2})}{9}$ (6)

Substituting (5) and (6) in (4) and applying the method of factorization, define

$$a + i\sqrt{2}t = \left(u + i\sqrt{2}v\right)^2 \frac{(1+i2\sqrt{2})}{3}$$

Equating the real and imaginary parts, we get

$$a(u, v) = \frac{1}{3}(u^2 - 2v^2 - 8uv)$$

$$t(u, v) = \frac{1}{3}(2u^2 - 4v^2 + 2uv)$$

As our interest centers on finding integer solutions, we choose suitable values for u and v so that the values of a and t are integers.

Let $= 3U$, $v = 3V$. Then

$$\left.\begin{aligned} a(U, V) &= (3U^2 - 6V^2 - 24UV) \\ t(U, V) &= (6U^2 - 12V^2 + 6UV) \end{aligned}\right\} \quad (7)$$

Using (5) and (7) in (2), the integral solutions to (1) are obtained as

$$x(U, V) = 9U^2 + 18V^2 + 1$$
$$y(U, V) = 9U^2 + 18V^2 - 1$$
$$z(U, V) = 3U^2 - 6V^2 - 24UV + 1$$
$$w(U, V) = 3U^2 - 6V^2 - 24UV - 1$$
$$t(U, V) = 6U^2 - 12V^2 + 6UV.$$

Properties

i. $t\big(U, U(U+1)\big) - 2w\big(U, U(U+1)\big) - 108P_U^5$ is a Kynea prime.
ii. $t(U(U+1), U) - x(U(U+1), U) + 24f_{4,5}^U - 20P_U^5 - 4t_{3,U} + 1$ is a nasty number.
iii. $t(\alpha^2 V, V) - 2w(\alpha^2 V, V) - 2$ is a nasty number.

B. Pattern II

Instead of (6) , write 1 as $1 = \frac{(7+i4\sqrt{2})(7-i4\sqrt{2})}{81}$ Following the procedure similar to pattern I and performing few

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calculations, the corresponding non-zero distinct integral solutions to (1) are given by

$$x(U,V) = 9U^2 + 18V^2 + 1$$
$$y(U,V) = 9U^2 + 18V^2 - 1$$
$$z(U,V) = 7U^2 - 14V^2 - 16UV + 1$$
$$w(U,V) = 7U^2 - 14V^2 - 16UV - 1$$
$$t(U,V) = 4U^2 - 8V^2 + 14UV$$

Properties

i. $z\big(U,U(U+1)\big) + w\big(U,U(U+1)\big) - t\big(U,U(U+1)\big) = 10t_{4,U} - 120f_{4,6}^{U} - 52P_U^5$.

ii. $t(1,V(V+1)) + t_{18,V(V+1)} - 7P(V)$ is a perfect square.

iii. $(1,V) - z(1,V) - 32t_{3,V}$ is a perfect square.

C. Pattern III

Instead of (6) , one may write 1 as $1 = \frac{(23+i10\sqrt{2})(23-i10\sqrt{2})}{729}$

Following the procedure similar to pattern I and performing few calculations, the corresponding non-zero distinct integral solutions to (1) are given by

$$x(U,V) = 81U^2 + 162V^2 + 1$$
$$y(U,V) = 81U^2 + 162V^2 - 1$$
$$z(U,V) = 69U^2 - 138V^2 - 120UV + 1$$
$$w(U,V) = 69U^2 - 138V^2 - 120UV - 1$$
$$t(U,V) = 30U^2 - 60V^2 + 138UV$$

Properties

i. $t(U(U+1),1) - z(U(U+1),1) - w(U(U+1),1) - 216t_{3,U(U+1)-1} - 270P(U)$ is a cubic integer

ii. $y(U(U+1),1) + z(U(U+1),1) - 300t_{3,U(U+1)-1} - 30P(U)$ is a nasty number.

iii. $y(U(U+1),1) - 162P_u^5 - 81CP_U^6 - 161$ is a biquadratic integer.

III. Conclusion

To conclude, one may search for other patterns of solutions and their corresponding properties.

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