IJISM Source of Knowledge

International Journal of Innovation in Science and Mathematics
Volume 2, Issue 4, ISSN (Online): 2347–9051

On Algebraic Properties of Soft Multiset Operations and Inclusion Relation

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***Abstract* – With the integration of the concept of multisets in soft multiset theory in the literature, it is imperative that the algebraic properties of multisets be examined in the light of Soft multiset theory. To this end, a comprehensive study of these algebraic properties such as Commutative, Associative, Idempotent, Distributive, absorbtion and Demorgan's laws are carried out. Though in general under this concept, the axiom of contradiction is satisfied with the exception of exclusion, we have established condition under which both are satisfied.**

***Keywords* – Multiset, Soft Multiset Commutative, Associative, Idempotent, Distributive, Absorbtion.**

I. Introduction

Many problems in engineering, Natural science, Social science, Medical science etc. involving uncertainties, cannot be handled with the available theories such as Set Theory, Fuzzy Set Theory, Probability Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. As earlier pointed in [3], these theories have certain limitations. In an attempt at removing those limitations, the notion of Soft Set Theory has been introduced with its fundamental results. In line with this theory Maji et al. [2] studied further the theory of soft sets and put forward some results. However, the axioms of exclusion and contradiction are not valid under the definition of complement of a soft set. In this regard, Neog and Sut [4] redefined the concept of complement of a soft set and established the laws of exclusion and contradiction, Involution, Demorgan inclusion and Demorgan's laws.

As a generalization of soft set in [3], the concept of Soft Multiset has been introduced in [1] with the laws of contradiction and exclusion invalid by its complement. However, in an attempt at removing this limitation, the concept of complement has been redefined in [5], satisfying the laws of exclusion and contradiction, Involution and Demorgan's laws. To integrate the concept of multiset in soft multiset theory, a soft multiset and some operations are redefined in [7] even though some algebraic properties of multiset concepts, operations, and inclusion are not studied under this integration . In this paper, we present a detailed study of algebraic properties of these concepts under Multisets using the new definition of soft multisets. Consequently, we redefine an empty (null) soft multiset, Universal soft multiset, restricted sum of soft multisets, difference of soft multisets and complement of a soft multiset. From these definitions, we deduce the algebraic properties such as Commutative, Associative, Idempotent, Distributive, absorbtion and Demorgan's laws. Though in general under this concept, the axiom of contradiction is satisfied with the exception of exclusion, we have established condition under which both are satisfied. Thus the paper is organized:In section 2, we present some basics on multisets, soft sets and propositions required in the subsequent sections of the paper . In section 3, we present some basic definitions and operations on soft multisets and deduce some of their algebraic properties.

II. Preliminaries

In this section, we recall the basic concept and notations of soft sets, multisets, operations and some algebraic properties on multisets with some existing results in the literature for subsequent referencing.

2.1 Soft set

In the current subsection, we recollect the basic definitions and notations as reintroduced in [6] and [5].

Definition 2.1. Let U be a universal set and E be a set of parameters. Let $\wp(U)$ denotes the power set of U and $A \subseteq E$. A pair (F,A) is called a soft set over U, where F is a mapping given by $F: A \to \wp(U)$.

Notes that $F(e) = \varnothing \quad \forall e \notin A$

Definition 2.2. For two soft sets (F,A) and (G,B) over a common universe U, (F,A) is a soft subset of (G,B) denoted $(F,A) \tilde{\subseteq} (G,B)$ if (i) $A \subseteq B$ and (ii) $F(e) \subseteq G(e) \ \forall e \in A$.

Definition 2.3. Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal denoted $(F,A) \tilde{=} (G,B)$ if $A = B$ and $F(e) = G(e) \ \forall e \in A$. In particular, $(F,A) = (G,B)$ if $(F,A) \tilde{\subseteq} (G,B)$ and $(G,B) \tilde{\subseteq} (F,A)$

Definition 2.4. Let (F,A) and (G,B) be soft sets over a common universe U. The union of soft sets (F,A) and (G,B) denoted $(F,A) \tilde{\bigcup} (G,B)$ is the soft set (H,C) where $C = A \bigcup B$ and $H = F \bigcup G$ such that

$H(e) = (F \bigcup G)(e) = F(e) \bigcup G(e)$ for all $e \in A \bigcup B$

Definition 2.5 [8]. Let (F,A) and (G,B) be soft sets over a common universe U. The intersection of soft sets (F,A) and (G,B) denoted $(F,A)\tilde{\cap}(G,B)$ is the soft set $(F\cap G, A\cap B)$ such that for all $e\in A\cap B, F(e)\cap G(e)\neq\varnothing \wedge (F\cap G)(e)=F(e)\cap G(e)$

In particular, if $A\cap B=\varnothing$, then $F\cap G=\varnothing$ and $<F\cap G, A\cap B> = <\varnothing,\varnothing>$

Definition 2.6 [7] . Let (F,A) be a soft set. The cardinality of (F,A) denoted $|(F,A)|$ is defined: $|(F,A)|=\sum_{e\in A}|F(e)|$

Proposition 2.1. If (F,A) and (G,B) are soft sets over a common universe U then

(i) $|(F,A)\tilde{\cup}(G,B)|$

$=|(F,A)|+|(G,B)|-|(F,A)\tilde{\cap}(G,B)|$

(ii) $(F,A)\tilde{\subseteq}(G,B)\rightarrow|(F,A)|\leq|(G,B)|$

Proof:

Let $(H,C)=(F,A)\tilde{\cup}(G,B)$

$|(H,C)|=\sum_{e\in A\cup B}|H(e)|=\sum_{e\in A\cup B}|F(e)\cup G(e)|$

But $\sum_{e\in A\cup B}|F(e)\cup G(e)|=\sum_{e\in A-B}|F(e)\cup G(e)|$

$+\sum_{e\in B-A}|F(e)\cup G(e)|+\sum_{e\in A\cap B}|F(e)\cup G(e)|$

$=\sum_{e\in A-B}|F(e)|+\sum_{e\in B-A}|G(e)|+\sum_{e\in A\cap B}|F(e)\cup G(e)|$

and $\sum_{e\in A\cap B}|F(e)\cup G(e)|$

$=\sum_{e\in A\cap B}|F(e)|+\sum_{e\in A\cap B}|G(e)|-\sum_{e\in A\cap B}|F(e)\cap G(e)|$

Thus, $\sum_{e\in A-B}|F(e)|+\sum_{e\in B-A}|G(e)|+\sum_{e\in A\cap B}|F(e)\cup G(e)|$

$=\sum_{e\in A-B}|F(e)|+\sum_{e\in B-A}|G(e)|+\sum_{e\in A\cap B}|F(e)|+\sum_{e\in A\cap B}|G(e)|$

$-\sum_{e\in A\cap B}|F(e)\cap G(e)|$

But $\sum_{e\in A-B}|F(e)|+\sum_{e\in A\cap B}|F(e)|$

$=\sum_{e\in A}|F(e)|, \sum_{e\in A\cap B}|G(e)|+\sum_{e\in B-A}|G(e)|=\sum_{e\in B}|G(e)|$

Hence, $\sum_{e\in A\cup B}|F(e)\cup G(e)|$

$=\sum_{e\in A}|F(e)|+\sum_{e\in B}|G(e)|-\sum_{e\in A\cap B}|F(e)\cap G(e)|$

In particular, $|(F,A)\tilde{\cup}(G,B)|$

$=|(F,A)|+|(G,B)|-|(F,A)\tilde{\cap}(G,B)|$

(ii) $(F,A)\tilde{\subseteq}(G,B)\rightarrow A\subseteq B\wedge F(e)\subseteq G(e)$.

But $F(e)\subseteq G(e)\rightarrow|F(e)|\leq|G(e)|\rightarrow\sum_{e\in A}|F(e)|\leq\sum_{e\in B}|G(e)|$

Thus, $(F,A)\tilde{\subseteq}(G,B)\rightarrow\sum_{e\in A}|F(e)|\leq\sum_{e\in B}|G(e)|$

In particular, $(F,A)\tilde{\subseteq}(G,B)\rightarrow|(F,A)|\leq|(G,B)|$

2.2 Multisets

Definition 2.8 [9]. A collection of elements which may contain duplicates is called a multiset (mset for short). Formally if X is a set of elements, a mset M drawn from the set X is represented by a count function C_M defined as $C_M: X\rightarrow\mathbb{N}$ where $\mathbb{N}$ represents the set of nonnegative integers. For each $x\in X$, $C_M(x)$ is the characteristic value of x in M indicating the number of occurrences of the element x in M called its multiplicity. An element $x\in X$ is an element of M if $C_M(x)>0$ or otherwise. A mset M is a set if $C_M(x)=0$ or 1 for all $x\in X$. A mset M is called null or empty mset denoted $\varnothing$ if $C_{\varnothing}(x)=0$ for all $x\in X$. For any mset M, it's root set (support set) denoted M^* is defined: $M^*=\{x\in X\,|C_M(x)>0\}$. Note that $\varnothing^*=\varnothing$.

The sum of all the multiplicities of M is called it's cardinality. We denote the cardinality of a mset M by $|M|$ and is defined: $|M|=\sum_{x\in M^*}C_M(x)$. A mset M is called finite if $|M|<\infty$. For any mset M, the predicate $set(M)$ is defined: $M=\varnothing\vee C_M(x)=n\rightarrow n=1$ for all $x\in X$.

In this paper, we assume finite msets. For the sake of convenience, a mset M is given by $M=\{k_1/x_1, k_2/x_2,\ldots,x_n/k_n\}$ in which the element x_i occurs k_i times where $k_i>0$.

We denote a finite class of finite msets over a finite set X by $\mathrm{M}_n^t(X)$ defined:

$$\mathrm{M}_n^t(X)=\left\{\begin{array}{l} M_1,M_2,\ldots,M_n\,|M_i\neq\varnothing\wedge M_i\ = \\ \{m_1/x_1,m_2/x_2,\ldots,m_k/x_k\}\wedge X \\ =\{x_1,x_2,\ldots,x_k\}\wedge m_j\in\{0,1,2,\ldots,t\} \\ \wedge i\in[1,n]\wedge j\in[1,k] \end{array}\right\}$$

Definition 2.9 [10]. Two msets $M,N\in\mathrm{M}_n^t(X)$ are said to be equal denoted $M=N$ if and only if $C_M(x)=C_N(x)\ \forall x\in X$

Definition 2.10 [10]. An mset $M\in\mathrm{M}_n^t(X)$ is a submultiset (submset for short) of a mset $N\in\mathrm{M}_n^t(X)$ denoted $M\subseteq N$ if $C_M(x)\leq C_N(x)$ for all $x\in X$. Note that $M\subseteq N\wedge N\subseteq M\leftrightarrow M=N$

and $C_M(x) \le C_N(x)$

$\to \min\{C_M(x), C_N(x)\}$

$= C_M(x) \wedge \max\{C_M(x), C_N(x)\}$

$= C_N(x)$

Also, $\varnothing \subseteq M$

since $C_\varnothing(x) = 0 \le C_M(x)\ \forall M \in \mathrm{M}_n^t(X)$

Definition 2.11. A mset $M \in \mathrm{M}_n^t(X)$ is a whole submset of a mset $N \in \mathrm{M}_n^t(X)$ denoted $M \subseteq N$ if (i) $M = \varnothing$ or (ii) $M \ne \varnothing$ and $C_M(x) = C_N(x)$ for all $x \in X$.

Definition 2.13 [9]. For any $M \in \mathrm{M}_n^t(X)$, the power mset of M denoted $\wp(M)$ is defined: $\wp(M) = \{A \mid A \subseteq M\}$ and set($\wp(M)$). i.e the power mset of any mset is defined to be the set of all submsets of the mset

Notes that $\varnothing, M \in \wp(M)$.

Definition 2.14[11] . For any $M \in \mathrm{M}_n^t(X)$, the power whole mset of M denoted $\wp w(M)$ is defined to be the set of all whole submsets of M.

Notes that $\varnothing, M \in \wp w(M)$

Definition 2.15. A non empty mset $U \in \mathrm{M}_n^t(X)$ is said to be a universal mset if $M \subseteq U$ for all $M \in \mathrm{M}_n^t(X)$

Theorem 2.2[9]. For any msets $M, N \in \mathrm{M}_n^t(X)$, we have

(i). $M \subseteq N \to M^* \subseteq N^*$ and

(ii) $M \subseteq N \to |M| \le |N|$

(iii) $M \subseteq N \wedge N \subseteq M \to M = N$

Definition 2.16 [11].

(i). The union of the msets $M, N \in \mathrm{M}_n^t(X)$ is the mset $A \in \mathrm{M}_n^t(X)$ denoted $A = M \cup N$ such that

$$C_A(x) = C_{M \cup N}(x) = \max\{C_M(x), C_N(x)\}$$

for all $x \in X$.

(ii). The intersection of the msets $M, N \in \mathrm{M}_n^t(X)$ is the mset $A \in \mathrm{M}_n^t(X)$ denoted $A = M \cap N$ such that $C_A(x) = C_{M \cap N}(x) = \min\{C_M(x), C_N(x)\}$ for all $x \in X$.

(iii).The difference of the msets $M, N \in \mathrm{M}_n^t(X)$ is the mset $A \in \mathrm{M}_n^t(X)$ denoted $A = M - N$ such that $C_A(x) = C_{M-N}(x) = \max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$.

(iv). The addition of the msets $M, N \in \mathrm{M}_n^t(X)$ is the mset $A \in \mathrm{M}_n^t(X)$ denoted $A = M + N$ such that $C_A(x) = C_{M+N}(x) = \min\{C_U(x), C_M(x) + C_N(x)\}$ for all $x \in X$ and U a universal mset

(v). The complement of the mset $M \in \mathrm{M}_n^t(X)$ denoted M^c is defined: $M^c = U - M$ where U is a universal mset.

Proposition 2.3. For any msets $M, N, P \in \mathrm{M}_n^t(X)$, we have :

(i) $M \cup (M + N) = M + N$

(ii) $M \cap (M + N) = M$

(iii) $M + N = (M \cup N) + (M \cap N)$

(iv) $M + (N + P) = (M + N) + P$

(v) $M + (N \cup P) = (M + N) \cup (M + P)$

(vi) $M + (N \cap P) = (M + N) \cap (M + P)$

(vii) $M + \varnothing = M$

Proof:

Now for any $x \in X$, we have

(i) $(M \cup (M + N))(x) = \max\{M(x), (M + N)(x)\}$

$= \max\{M(x), \max\{U(x), M(x) + N(x)\}\}$

Since $M(x) \le \max\{U(x), M(x) + N(x)\}$, we have

$\max\{M(x), \max\{U(x), M(x) + N(x)\}\}$

$= \max\{U(x), M(x) + N(x)\} = (M + N)(x)$

In particular, $(M \cup (M + N))(x) = (M + N)(x)$

and $M \cup (M + N) = M + N$

(ii) $(M \cap (M + N))(x) = \min\{M(x), (M + N)(x)\}$

$= \min\{M(x), \max\{U(x), M(x) + N(x)\}\}$

Since $M(x) \le \max\{U(x), M(x) + N(x)\}$, we have

$\min\{M(x), \max\{U(x), M(x) + N(x)\}\} = M(x)$

In particular, $(M \cap (M + N))(x) = M(x)$

and $M \cap (M + N) = M$

(iii). $C_{M+N}(x) = \min\{C_U(x), C_M(x) + C_N(x)\}$

$$= \min\left\{\begin{matrix} C_U(x), \max\{C_M(x), C_N(x)\} \\ + \min\{C_M(x), C_N(x)\} \end{matrix}\right\}$$

$= \min\{C_U(x), C_{M \cup N}(x) + C_{M \cap N}(x)\}$

$= C_{(M \cup N)+(M \cap N)}(x).$

Thus, $M + N = (M \cup N) + (M \cap N)$

(iv) $$C_{M+(N+P)}(x) = \min\left\{\begin{matrix} C_U(x), \\ C_M(x) + C_{N+P}(x) \end{matrix}\right\}.$$

$$= \min\left\{\begin{matrix} C_U(x), C_M(x) \\ + \min\{C_U(x), C_N(x) + C_P(x)\} \end{matrix}\right\}$$

But $$\min\left\{\begin{matrix} C_U(x), C_M(x) \\ + \min\{C_U(x), C_N(x) + C_P(x)\} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} C_U(x), \\ \min\{C_U(x), C_M(x) + C_N(x)\} + C_P(x) \end{matrix}\right\}$$

International Journal of Innovation in Science and Mathematics
Volume 2, Issue 4, ISSN (Online): 2347–9051

i.e $C_{M+(N+P)}(x)=\min\{C_U(x),C_M(x)+C_{N+P}(x)\}$

$=\min\{C_U(x),C_{M+N}(x)+C_P(x)\}$

$=C_{(M+N)+P}(x)$

i.e $M+(N+P)=(M+N)+P$

(v) $C_{M+(N\cup P)}(x)$

$=\min\{C_U(x),C_M(x)+\max\{C_N(x),C_P(x)\}\}$

$=\max\begin{Bmatrix}\min\{C_U(x),C_M(x)+C_N(x)\},\\ \min\{C_U(x),C_M(x)+C_P(x)\}\end{Bmatrix}$

$=\max\{C_{M+N}(x),C_{M+P}(x)\}$

$=C_{(M+N)\cup(M+P)}(x)$

i.e $M+(N\cup P)=(M+N)\cup(M+P)$

(vi) $C_{M+(N\cap P)}(x)=\min\begin{Bmatrix}C_U(x),C_M(x)+\\ \min\{C_N(x),C_P(x)\}\end{Bmatrix}$

$=\min\begin{Bmatrix}\min\{C_U(x),C_M(x)+C_N(x)\},\\ \min\{C_U(x),C_M(x)+C_P(x)\}\end{Bmatrix}$

$=\min\{C_{M+N}(x),C_{M+P}(x)\}$

$=C_{(M+N)\cap(M+P)}(x)$

i.e $M+(N\cap P)=(M+N)\cap(M+P)$

(vii) $C_{M+\varnothing}(x)=\min\{C_U(x),C_M(x)+C_\varnothing(x)\}$

$=\min\{C_U(x),C_M(x)+0\}$

$=\min\{C_U(x),C_M(x)\}=C_M(x)$

i.e $M+\varnothing=M$

Theorem 2.4. For any msets $M,N\in M_n^t(X)$ such that $M\subseteq N$, we have:

$N-(N-M)=M$

Proof:

$C_{N-(N-M)}(x)=\max\{C_N(x)-C_{N-M}(x),0\}$

$=\max\begin{Bmatrix}C_N(x)-\max\{C_N(x)-C_M(x),0\}\\ ,0\end{Bmatrix}$ (by definition)

But $\max\{C_N(x)-C_M(x),0\}=C_N(x)-C_M(x)$ (since $M\subseteq N$)

Thus,

$C_{N-(N-M)}(x)=\max\{C_N(x)-C_{N-M}(x),0\}$

$=\max\{C_N(x)-(C_N(x)-C_M(x)),0\}=C_M(x)$

In particular, $N-(N-M)=M$

Theorem 2.5 [10]. For any msets , we have

(i). $\left(\bigcup_{i=1}^{r}M_i\right)^*=\bigcup_{i=1}^{r}M_i^*$

(ii). $\left(\bigcap_{i=1}^{r}M_i\right)^*=\bigcap_{i=1}^{r}M_i^*$

Theorem 2.6 [12]. For any msets $M,N\in M_n^t(X)$, we have

$|M\cup N|=|M|+|N|-|M\cap N|$.

Theorem 2.6 ([10],[12]). For any msets $M,N,P\in M_n^t(X)$, we have:

(i) $M\cup M=M$

(ii) $M\cap M=M$

(iii) $M\cup N=N\cup M$

(iv) $M\cap N=N\cap M$

(v) $M\cup(N\cup P)=(M\cup N)\cup P$

(vi) $M\cap(N\cap P)=(M\cap N)\cap P$

(vii) $(M^c)^c=M$

(viii). $\varnothing^c=U$

(ix) $U^c=\varnothing$

(x) $(M\cup N)^c=M^c\cap N^c$

(xi). $(M\cap N)^c=M^c\cup N^c$

(xii). $M\cup(N\cap P)=(M\cup N)\cap(M\cup P)$

(xiii). $M\cap(N\cup P)=(M\cap N)\cup(M\cap P)$

Note that $M,N\in M_n^t(X)$ such that $M\subseteq N$ implies $M\cap N=M$ and $M\cup N=N$

Since $C_M(x)\le\max\{C_M(x),C_N(x)\}$

$\wedge\min\{C_M(x),C_N(x)\}\le C_M(x)$

we have $\min\{C_M(x),\max\{C_M(x),C_N(x)\}\}=C_M(x)$

and $\max\{\min\{C_M(x),C_N(x)\},C_M(x)\}=C_M(x)$.

Thus, we have $M\cap(M\cup N)=M$

and $M\cup(M\cap N)=M$

In general, for any $M,N\in M_n^t(X)$,

it can be easily proved that

(i) $M\cup(M\cap N)=M$ (ii) $M\cap(M\cup N)=M$

Also, $\min\{C_M(x),C_N(x)\}\le\max\{C_M(x),C_N(x)\}$

$\le\min\{C_U(x),C_M(x)+C_N(x)\}$

Thus we have $C_{M\cap N}(x)\le C_{M\cup N}(x)\le C_{M+N}(x)$

and $M\cap N\subseteq M\cup N\subseteq M+N$

III. Soft Multisets

The idea of soft multisets was first introduced in [1]. Majumdar [7] redefined soft multiset. Also, in the light of this definition redefined the union, intersection ,restricted sum and Cartesian product operations and established some algebraic properties of the union and intersection such as commutativity, idempotency, associativity and distributivity.

In this section, we redefined the soft multiset and introduce new concepts. The restricted sum operation is redefined and more of algebraic properties of the restricted sum and Cartesian product are established.

Definition 3.1. Let $U \in M_n^t(X)$ be a universal mset, E be set of parameters and $A \subseteq E$. Then a triple $< F, A, C_F >$ characterized by its soft count function $C_F : A \to \mathbb{N}^X$ defined: $C_F(e) = c_F^e \in \mathbb{N}^X$ where $c_F^e : X \to \mathbb{N}$ is the parameterized count function with $\mathbb{N}$ the set of non negative integers and $F : A \to \wp(U)$ is defined such that corresponding to each $e \in E$, every element $x \in X$ occurs exactly $c_F^e(x)$ times in $F(e)$ is called a soft multiset (soft mset for short).

Note that any soft set $< F, A, >$ is a soft mset $< F, A, C_F >$ *in which* $C_F^e(x) \in \{0,1\}\ \forall e \forall x (e \in A \wedge x \in X)$

Example 3.1 Let U be a universal set over set of English alphabets given by $U = \{5/x, 3/y, 4/z, 6/w\}$ and $E = \{1,2,3\}$.

Define a mapping $F : E \to \wp(U)$ as follows:

$F(1) = \{1/x, 2/y, 3/z\}$, $F(2) = \{3/x, 1/y, 2/z, 3/w\}$,

$F(3) = \{4/x, 3/y, 3/z, 2/w\}$

Then $< F, A, C_F >$ is a soft mset where $\forall e \in A$, $F(e)$ mset is represented by count function $C_F^e : X \to \mathbb{N}$ defined:

$C_F^1(x) = 1, C_F^1(y) = 2, C_F^1(z) = 3$

$C_F^2(x) = 3, C_F^2(y) = 1, C_F^2(z) = 2, C_F^2(w) = 3$

$C_F^3(x) = 4, C_F^3(y) = 3, C_F^3(z) = 3, C_F^3(w) = 2$

Then $< F, A, C_F > = \{F(1), F(2), F(3)\}$

$$= \begin{Bmatrix} \{1/x, 2/y, 3/z\}, \{3/x, 1/y, 2/z, 3/w\} \\ , \{4/x, 3/y, 3/z, 2/w\} \end{Bmatrix}$$

Note that for a soft mset $< F, A, C_F >$, $F(e) = \varnothing \leftrightarrow e \notin A$

Definition 3.1

Let the mset $U \in M_n^t(X)$ be universal, E be the set of parameters and $A \subset E$. Then a soft mset $< F, A, C_F >$ is said to be null (empty) if and only if $A = \varnothing$. Symbolically, we write $(\varnothing, \varnothing, \varnothing)$ for the empty soft mset over U.

Definition 3.2. A non empty soft mset $< F, A, C_F >$ over the universal mset $U \in M_n^t(X)$ is said to be absolute if $F(e) = U$ for all $e \in A$. In this case, $C_F^e(x) = C_U(x)\ \forall e \in A, \forall x \in X$. We denote the absolute mset by $< F, A, C_F >_U$

Definition 3.3 [7]. Let $< F, A, C_F >$ be a soft mset over the universal mset $U \in M_n^t(X)$. The cardinality of $< F, A, C_F >$ denoted $|< F, A, C_F >|$ is defined:

$$|< F, A, C_F >| = \sum_{e \in A} |F(e)| = \sum_{e \in A} \sum_{x \in U} C_F^e(x).$$

Example 3.2 From example 3.1 above, we have: $|F(1)| = 6, |F(2)| = 9, |F(3)| = 12.$

$$\therefore \sum_{e \in A} |F(e)| = |F(1)| + |F(2)| + |F(3)| = 6 + 9 + 12 = 27$$

In particular, $|< F, A, C_F >| = 27$

Definition 3.4 Let $< F, A, C_F >, < G, B, C_G >$ be a soft msets over the universal mset $U \in M_n^t(X)$. Then $< F, A, C_F >$ is equal to $< G, B, C_G >$ denoted $< F, A, C_F > \tilde{=} < G, B, C_G >$ if and only if (i) $A = B$ (ii) $F(e) = G(e)$ for all $e \in A, B$. i.e $C_F^e(x) = C_G^e(x)\ \ \forall x \in X, \forall e \in A, B$.

Definition 3.5 . Let $< F, A, C_F >, < G, B, C_G >$ be a soft msets over the universal mset $U \in M_n^t(X)$. The soft mset $< F, A, C_F >$ is a soft multi subset of $< G, B, C_G >$ denoted $< F, A, C_F > \tilde{\subseteq} < G, B, C_G >$ if and only if (i) $< F, A, C_F > = < \varnothing, \varnothing, \varnothing >$ or (ii) $A \subseteq B$ and $F(e) \subseteq G(e)$ for all $e \in A$. i.e $C_F^e(x) \le C_G^e(x)\ \ \forall x \in X, \forall e \in A$.

Notes that the empty soft mset $< \varnothing, \varnothing, \varnothing >$ is a soft multi subset of any soft multiset by definition. Also,

$< F, A, C_F > \tilde{\subseteq} < G, B, C_G >$

$\wedge < G, B, C_G > \tilde{\subseteq} < F, A, C_F >$

$\to < F, A, C_F > \tilde{=} < G, B, C_G >$

Since $A \subseteq B \wedge B \subseteq A \to A = B$ *and*

$F(e) \subseteq G(e) \wedge G(e) \subseteq F(e)$

$\to F(e) = G(e)\ \forall e \in A = B \subseteq E$

Definition 3.6. Let $< F, A, C_F >$ be a soft msets over the universal mset $U \in M_n^t(X)$.

The soft mset $< F, A, C_F >$ is said to be a whole soft mset if $F(e)$ *is a whole submset of* $U\ \ \forall e \in A$.

Definition 3.7

A soft mset $< G, B, C_G >$ over the universal mset $U \in M_n^t(X)$ is said to be universal if and only if $< G, B, C_G >$ is absolute and $\varnothing \neq B = E$. We denote a universal soft mset by $< \mu, E, C_\mu >$.

Notes that a universal soft mset is unique since having $< G, B, C_G >$ as another universal soft mset results into $< G, B, C_G > \tilde{=} < \mu, E, C_\mu >$.

Definition 3.8. Let $U \in M_n^t(X)$ be a universal mset, E be set of parameters and $A \subseteq E$. Then the root soft set of a soft mset

$< F, A, C_F >$ denoted $(< F, A, C_F >)^*$

is defined:

$(< F, A, C_F >)^* = (F^*, A)$ such that

$F^* : A \to \wp(U^*)$ is defined:

$F^*(e) = (F(e))^* \ \forall e \in A.$

Notes that (F^*, A) is a soft set over the root set of the universal mset U

Definition 3.9 .Let $< F,A,C_F >, < G, B, C_G >$ be a soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B \subseteq E.$

The following operations are defined

(i) *Union*

$< F,A,C_F > \tilde{\bigcup} < G, B, C_G >$

$\cong < F \bigcup G, A \bigcup B, C_{F \bigcup G} >$ *such that*

$(F \bigcup G)(e) = F(e) \bigcup G(e)$ for all $e \in A \bigcup B$

(ii)Intersection

$< F,A,C_F > \tilde{\bigcap} < G, B, C_G >$

$\cong < F \bigcap G, A \bigcap B, C_{F \bigcap G} >$ such that

$(F \bigcap G)(e) = F(e) \bigcap G(e) \quad e \in A \bigcap B$

and $C^e_{F \bigcap G}(x) = \min\{C^e_F(x), C^e_G(x)\}$

Notes that if $A \bigcap B = \varnothing$ then

$< F,A,C_F > \tilde{\bigcap} < G, B, C_G > \cong < \varnothing, \varnothing, \varnothing >$

(iii) Restricted sum

$< F,A,C_F > \oplus < G, B, C_G >$

$= < F + G, A \bigcup B, C_{F+G} >$ such that

$(F + G)(e) = F(e) + G(e)$ for all $e \in A \bigcup B$

and $C^e_{F+G}(x)$

$= \min\{C_U(x), C^e_F(x) + C^e_G(x)\}$

(iv) Difference

$< F,A,C_F > \Theta < G, B, C_G >$

$= < F - G, A - B, C_{F-G} >$ such that

$(F - G)(e) = F(e) - G(e)$

for all $e \in A - B$

(v) *Complement*

The complement of $< F, A, C_F)$

denoted $(< F, A, C_F))^c$ is defined:

$(< F, A, C_F))^c = < \mu, E, C_\mu > \Theta < F, A, C_F)$

Note that $< \mu, E, C_\mu > \Theta < F, A, C_F)$

$= < \mu - F, E - A, C_{\mu - F} >$

We denote F^c for $\mu - F$ and $E - A = A'.$

Thus, $< \mu, E, C_\mu > \Theta < F, A, C_F >$

$= < \mu - F, E - A, C_{\mu - F} > = < F^c, A', C_{F^c} >$

In particular, $(< F, A, C_F >)^c$

$= < F^c, A', C_{F^c} >$ where

$F^c(e) = \mu(e) - F(e)$ for all $e \in A'$

Proposition 3.1. Let $< F,A,C_F >, < G, B, C_G >$ be a soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B \subseteq E$. Then

(i) $(< F,A,C_F > \tilde{\bigcup} < G, B, C_G >)^*$

$\cong (< F,A,C_F >)^* \tilde{\bigcup} (< G, B, C_G >)^*$

(ii) $(< F,A,C_F > \tilde{\bigcap} < G, B, C_G >)^*$

$\cong (< F,A,C_F >)^* \tilde{\bigcap} (< G, B, C_G >)^*$

Proof:

(i) $(< F,A,C_F > \tilde{\bigcup} < G, B, C_G >)^*$

$\cong (< F \bigcup G, A \bigcup B, C_{F \bigcup G})^*$ (by definition)

$= ((F \bigcup G)^*, A \bigcup B)$ such that

$(F \bigcup G)^* : A \bigcup B \to \wp(U^*)$ defined:

$(F \bigcup G)^*(e) = ((F \bigcup G)(e))^*$

$= (F(e) \bigcup G(e))^*$

$= (F(e))^* \bigcup (G(e))^* = F^*(e) \bigcup G^*(e)$

$= (F^* \bigcup G^*)(e)$ for all $e \in A \bigcup B$

Thus, $(F \bigcup G)^* = F^* \bigcup G^*$

In particular, $(< F,A,C_F > \tilde{\bigcup} < G, B, C_G >)^*$

$\cong ((F \bigcup G)^*, A \bigcup B) = (F^* \bigcup G^*, A \bigcup B)$

$= (F^* A) \tilde{\bigcup} (G^*, B)$

$\cong (< F,A,C_F >)^* \tilde{\bigcup} (< G, B, C_G >)^*$

(ii) $(< F,A,C_F > \tilde{\bigcap} < G, B, C_G >)^*$

$\cong (< F \bigcap G, A \bigcap B, C_{F \bigcap G})^*$

$\cong ((F \bigcap G)^*, A \bigcap B)$

But $(F \bigcap G)^*(e) = ((F \bigcap G)(e))^*$

$= (F(e))^* \bigcap (G(e))^*$

$= F^*(e) \cap G^*(e)$ for $e \in A \cap B$

$= \left(F^* \cap G^*\right)(e)$

In particular, $\left(F \cap G\right)^* = F^* \cap G^*$

and $\left(< F \cap G, A \cap B, C_{F \cap G}\right)^*$

$\cong \left(\left(F \cap G\right)^*, A \cap B\right) \cong \left(F^* \cap G^*, A \cap B\right)$

But $\left(F^* \cap G^*, A \cap B\right) \cong \left(F^*, A\right) \tilde{\cap} \left(G^*, B\right)$

$\cong \left(< F, A, C_F >\right)^* \tilde{\cap} \left(< G, B, C_G >\right)^*$

Thus, $\left(< F, A, C_F > \tilde{\cap} < G, B, C_G >\right)^*$

$= \left(< F, A, C_F >\right)^* \tilde{\cap} \left(< G, B, C_G >\right)^*$

Proposition 3.2

Let $\left\{< F_i, A_i, C_{F_i} > \middle| i = 1, 2, ..., n\right\}$

be a finite class of soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A_i \subseteq E$. Then

(i) $\left(\tilde{\cup} < F_i, A_i, C_{F_i} >\right)^* = \tilde{\cup}\left(< F_i, A_i, C_{F_i} >\right)^*$

(ii) $\left(\tilde{\cap} < F_i, A_i, C_{F_i} >\right)^* = \tilde{\cap}\left(< F_i, A_i, C_{F_i} >\right)^*$

Proof:

This is clear and straight forward as consequents of proposition 3.1

Proposition 3.3 *Let* $< F, A, C_F >, < G, B, C_G >$ be soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B \subseteq E$. Then $< F, A, C_F > \tilde{\subseteq} < G, B, C_G >$ $\rightarrow \left(< F, A, C_F >\right)^* \tilde{\subseteq} \left(< G, B, C_G >\right)^*$.

Proof:

Assumming $< F, A, C_F > \tilde{\subseteq} < G, B, C_G >$,

then $A \subseteq B \wedge F(e) \subseteq G(e)$ *for all* $e \in A$

i.e $\left(F(e)\right)^* \subseteq \left(G(e)\right)^*$ *for all* $e \in A$.

In particular, $F^*(e) \subseteq G^*(e)$ *for all* $e \in A$

Thus, $\left(F^*, A\right) \tilde{\subseteq} \left(G^*, B\right)$.

In particular, $\left(< F, A, C_F >\right)^* \tilde{\subseteq} \left(< G, B, C_G >\right)^*$

Hence, $< F, A, C_F > \tilde{\subseteq} < G, B, C_G >$

$\rightarrow \left(< F, A, C_F >\right)^* \tilde{\subseteq} \left(< G, B, C_G >\right)^*$

Proposition 3.4. Let $< F, A, C_F >, < G, B, C_G >$ be a soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B \subseteq E$. Then

$< F, A, C_F > \tilde{\cong} < G, B, C_G >$

$\rightarrow \left(< F, A, C_F >\right)^* \cong \left(< G, B, C_G >\right)^*$

Proof:

$< F, A, C_F > \tilde{\cong} < G, B, C_G >$

$\rightarrow A = B,\ F(e) = G(e)$

and $\left(F(e)\right)^* = \left(G(e)\right)^*$

i.e $F^*(e) = G^*(e)$

and $< F^*(e), A > \tilde{\cong} < G^*(e), B >$

In particular, $\left(< F, A, C_F >\right)^* \cong \left(< G, B, C_G >\right)^*$

Proposition 3.5. *Let* $< F, A, C_F >, < G, B, C_G >$ be a soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B \subseteq E$. Then

(i) $\left|< F, A, C_F > \tilde{\cup} < G, B, C_G >\right|$

$= \left|< F, A, C_F >\right| + \left|< G, B, C_G >\right|$

$- \left|< F, A, C_F > \tilde{\cap} < G, B, C_G >\right|$

(ii) $< F, A, C_F > \tilde{\subseteq} < G, B, C_G >$

$\rightarrow \left|< F, A, C_F >\right| \leq \left|< G, B, C_G >\right|$

Pr*oof* :

(i) $\left|< F, A, C_F > \tilde{\cup} < G, B, C_G >\right|$

$= \sum_{e \in A \cup B} \left|F(e) \cup G(e)\right|$

$= \sum_{e \in A - B} \left|F(e)\right| + \sum_{e \in B - A} \left|G(e)\right| + \sum_{e \in A \cap B} \left|F(e) \cup G(e)\right|$ (1)

But $\sum_{e \in A \cap B} \left|F(e) \cup G(e)\right|$

$= \sum_{e \in A \cap B} \left|F(e)\right| + \sum_{e \in A \cap B} \left|G(e)\right| - \sum_{e \in A \cap B} \left|F(e) \cap G(e)\right|$

Thus,

$\sum_{e \in A \cup B} \left|F(e) \cup G(e)\right| = \sum_{e \in A - B} \left|F(e)\right| + \sum_{e \in B - A} \left|G(e)\right| +$

$\sum_{e \in A \cap B} \left|F(e)\right| + \sum_{e \in A \cap B} \left|G(e)\right| - \sum_{e \in A \cap B} \left|F(e) \cap G(e)\right|$ (2)

also, $\sum_{e \in A - B} \left|F(e)\right| + \sum_{e \in A \cap B} \left|F(e)\right| = \sum_{e \in A} \left|F(e)\right|,$

$\sum_{e \in B - A} \left|G(e)\right| + \sum_{e \in A \cap B} \left|G(e)\right| = \sum_{e \in B} \left|G(e)\right|$ (3)

Substituting (2) and (3) in (1) above, we have:

$\left|< F, A, C_F > \tilde{\cup} < G, B, C_G >\right|$

$= \sum_{e \in A} \left|F(e)\right| + \sum_{e \in B} \left|G(e)\right| - \sum_{e \in A \cap B} \left|F(e) \cap G(e)\right|$

In particular, $\left|< F, A, C_F > \tilde{\cup} < G, B, C_G >\right|$

$= \left|< F, A, C_F >\right| + \left|< G, B, C_G >\right|$

$- \left|< F, A, C_F > \tilde{\cap} < G, B, C_G >\right|$

(ii) Assuming that $< F, A, C_F > \tilde{\subseteq} < G, B, C_G >$,

then $A \subseteq B \wedge F(e) \subseteq G(e)$ *for all* $e \in A$.

Thus, $\sum_{e \in A} \left|F(e)\right| \leq \sum_{e \in A} \left|G(e)\right| \leq \sum_{e \in B} \left|G(e)\right|$.

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In particular, $\sum_{e\in A}|F(e)| \leq \sum_{e\in B}|G(e)|$.

Thus, $|<F,A,C_F>| \leq |<G,B,C_G>|$

whenever $<F,A,C_F> \tilde{\subseteq} <G,B,C_G>$.

Proposition 3.6. *Let* $<F,A,C_F>, <G,B,C_G>$ be a soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B \subseteq E$. Then

(i) $<F,A,C_F> \tilde{\cap} <G,B,C_G>$
$\tilde{\subseteq} <F,A,C_F> \tilde{\cup} <G,B,C_G>$
$\tilde{\subseteq} <F,A,C_F> \oplus <G,B,C_G>$

(ii) $<F,A,C_F> \oplus <G,B,C_G>$
$= \left(<F,A,C_F> \tilde{\cup} <G,B,C_G>\right)$
$\oplus \left(<F,A,C_F> \tilde{\cap} <G,B,C_G>\right)$

Proof:

(i) $<F,A,C_F> \tilde{\cap} <G,B,C_G>$
$\tilde{\cong} <F \cap G, A \cup B, C_{F\cap G}>$.

But $A \cap B \subseteq A \cup B$ and $(F \cap G)(e)$
$= F(e) \cap G(e) \subseteq F(e) \cup G(e)$ *for all* $e \in A \cup B$

Thus, $(F \cap G)(e) \subseteq (F \cup G)(e)$ *for all* $e \in A \cup B$.

and $<F \cap G, A \cap B, C_{F\cap G}> \tilde{\subseteq} <F \cup G, A \cup B, C_{F\cup G}>$.

In particular, $<F,A,C_F> \tilde{\cap} <G,B,C_G>$
$\tilde{\subseteq} <F,A,C_F> \tilde{\cup} <G,B,C_G>$ (1)

$<F,A,C_F> \tilde{\cup} <G,B,C_G>$
$\tilde{\cong} <F \cup G, A \cup B, C_{F\cup G}>$.

But $(F \cup G)(e) = F(e) \cup G(e)$ $e \in A \cup B$
(since $F(e) \cup G(e) \subseteq F(e) + G(e)$
for all $e \in A \cap B$ *and* $A \cap B \subseteq A \cup B$).

We have $F(e) \cup G(e) \subseteq F(e) + G(e)$
for all $e \in A \cup B$ In particular,
$<F \cup G, A \cup B, C_{F\cup G}>$
$\tilde{\subseteq} <F+G, A \cup B, C_{F+G}>$

i.e $<F,A,C_F> \tilde{\cup} <G,B,C_G>$
$\tilde{\subseteq} <F,A,C_F> \oplus <G,B,C_G>$ (2)

Since the relation $\subseteq$ is transitive,

we deduce the result from (1) and (2) :

$<F,A,C_F> \tilde{\cap} <G,B,C_G>$
$\tilde{\subseteq} <F,A,C_F> \tilde{\cup} <G,B,C_G>$
$\tilde{\subseteq} <F,A,C_F> \oplus <G,B,C_G>$

(ii)

$<F,A,C_F> \oplus <G,B,C_G>$
$\tilde{\cong} <F+G, A \cup B, C_{F+G}>$ where
$(F+G)(e) = F(e) + G(e)$ $e \in A \cup B$ (1)

But $F(e) + G(e)$
$= (F(e) \cup G(e)) + (F(e) \cap G(e))$.

Thus (1) is rewritten:

$(F+G)(e)$
$= (F(e) \cup G(e)) + (F(e) \cap G(e))$ $e \in A \cup B$
$= ((F \cup G) + (F \cap G))(e)$. *In particular*, $F+G$
$= (F \cup G) + (F \cap G)$

and $C_{F+G} = C_{(F\cup G)+(F\cap G)}$

Thus, $<F,A,C_F> \oplus <G,B,C_G>$
$\tilde{\cong} <F+G, A \cup B, C_{F+G}>$
$\tilde{\cong} <(F \cup G) + (F \cap G), A \cup B, C_{(F\cup G)+(F\cap G)}>$
$\tilde{\cong} <F \cup G, A \cup B, C_{F\cup G}> \oplus <F \cap G, A \cup B, C_{F\cap G}>$
$\tilde{\cong} \left(<F,A,C_F> \tilde{\cup} <G,B,C_G>\right)$
$\oplus \left(<F,A,C_F> \tilde{\cap} <G,B,C_G>\right)$

Proposition 3.7.

Let $<F,A,C_F>, <G,B,C_G>$ *and* $<H,C,C_H>$ be a soft msets over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B, C \subseteq E$. Then

(i) $<F,A,C_F> \oplus \left(<G,B,C_G> \tilde{\cup} <H,C,C_H>\right)$
$= (<F,A,C_F> \oplus <G,B,C_G>)$
$\tilde{\cup} (<F,A,C_F> \oplus <H,C,C_H>)$

(ii) $<F,A,C_F> \oplus \left(<G,B,C_G> \tilde{\cap} <H,C,C_H>\right)$
$= (<F,A,C_F> \oplus <G,B,C_G>)$
$\tilde{\cap} (<F,A,C_F> \oplus <H,C,C_H>)$

(iii) $<F,A,C_F> \oplus (<G,B,C_G> \oplus <H,C,C_H>)$
$= (<F,A,C_F> \oplus <G,B,C_G>) \oplus <H,C,C_H>$

Proof:

(i) $<F,A,C_F> \oplus \left(<G,B,C_G> \tilde{\cup} <H,C,C_H>\right)$
$\tilde{\cong} <F,A,C_F> \oplus <G \cup H, B \cup C, C_{G\cup H}>$
$\tilde{\cong} <F + (G \cup H), A \cup (B \cup C), C_{F+(G\cup H)}>$
such that $(F + (G \cup H))(e)$
$= F(e) + (G \cup H)(e)$ $e \in A \cup (B \cup C)$

But $F(e) + (G \cup H)(e)$
$= F(e) + (G(e) \cup H(e))$
$= (F(e) + G(e)) \cup (F(e) + H(e))$
$= (F+G)(e) \cup (F+H)(e)$
$= ((F+G) \cup (F+H))(e)$.

Thus, $F + (G \cup H) = (F+G) \cup (F+H)$ (1)

also, $A \cup (B \cup C) = (A \cup B) \cup (A \cup C)$ (2)

Hence, from (1) and (2) we have:

$<F + (G \cup H), A \cup (B \cup C), C_{F+(G\cup H)}>$
$\tilde{\cong} <(F+G) \cup (F+H)$,

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$(A\cup B)\cup(A\cup C),C_{(F+G)\cup(F+H)}>$
$=<F+G,A\cup B,C_{F+G}>$
$\tilde{\bigcup}<F+H,A\cup C,C_{F+H}>$
$=(<F,A,C_F>\oplus<G,B,C_G>)$
$\tilde{\bigcup}(<F,A,C_F>\oplus<H,C,C_H>)$

In particular,

$<F,A,C_F>\oplus\left(<G,B,C_G>\tilde{\bigcup}<H,C,C_H>\right)$
$=(<F,A,C_F>\oplus<G,B,C_G>)$
$\tilde{\bigcup}(<F,A,C_F>\oplus<H,C,C_H>)$

(ii) $<F,A,C_F>\oplus\left(<G,B,C_G>\tilde{\bigcap}<H,C,C_H>\right)$
$\cong<F,A,C_F>\oplus<G\cap H,B\cap C,C_{G\cap H}>$
$\cong<F+(G\cap H),A\cup(B\cap C),C_{F+(G\cap H)}>$

such that

$(F+(G\cap H))(e)$
$=F(e)+(G\cap H)(e)\ \ e\in A\cup(B\cap C)$

But $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ (1)

and $(F+(G\cap H))(e)=F(e)+(G\cap H)(e)$
$=F(e)+(G(e)\cap H(e))$
$=(F(e)+G(e))\cap(F(e)+H(e))$
$=(F+G)(e)\cap(F+H)(e)$
$=((F+G)\cap(F+H))(e).$

Thus, $F+(G\cap H)=(F+G)\cap(F+H)$ (2)

From (1) and (2) we have,

$<F+(G\cap H),A\cup(B\cap C),C_{F+(G\cap H)}>$
$\cong<(F+G)\cap(F+H),$
$(A\cup B)\cap(A\cup C),C_{(F+G)\cap(F+H)}>$
$\cong<F+G,A\cup B,C_{F+G}>$
$\tilde{\bigcap}<F+H,A\cup C,C_{F+H}>$
$\cong(<F,A,C_F>\oplus<G,B,C_G>)$
$\tilde{\bigcap}(<F,A,C_F>\oplus<H,C,C_H>).$

In particular,

$<F,A,C_F>\oplus\left(<G,B,C_G>\tilde{\bigcap}<H,C,C_H>\right)$
$\cong(<F,A,C_F>\oplus<G,B,C_G>)$
$\tilde{\bigcap}(<F,A,C_F>\oplus<H,C,C_H>).$

(iii) $<F,A,C_F>\oplus(<G,B,C_G>\oplus<H,C,C_H>)$
$\cong<F,A,C_F>\oplus<G+H,B\cup C,C_{G+H}>$
$\cong<F+(G+H),A\cup(B\cup C),C_{F+(G+H)}>$

such that

$(F+(G+H))(e)$
$=F(e)+(G+H)(e)\ \ e\in A\cup(B\cup C)$
$=F(e)+(G(e)+H(e))$
$=(F(e)+G(e))+H(e)$
$=(F+G)(e)+H(e)$
$=((F+G)+H)(e).$

Thus, $F+(G+H)=(F+G)+H$ (1)

But $A\cup(B\cup C)=(A\cup B)\cup C$ (2)

Hence from (1) and (2) we have:

$<F+(G+H),A\cup(B\cup C),C_{F+(G+H)}>$
$\cong<(F+G)+H,(A\cup B)\cup C,C_{(F+G)+H}>$
$\cong<F+G,A\cup B,C_{F+G}>\oplus<H,C,C_H>$
$\cong(<F,A,C_F>\oplus<G,B,C_G>)\oplus<H,C,C_H>$

In particular,

$<F,A,C_F>\oplus(<G,B,C_G>\oplus<H,C,C_H>)$
$\cong(<F,A,C_F>\oplus<G,B,C_G>)\oplus<H,C,C_H>$

Proposition 3.8 Let $<F,A,C_F>$, $<G,B,C_G>$ and $<H,C,C_H>$ be a soft msets over the universal mset $U\in M_n^t(X)$ and E be a set of parameters such that $A,B,C\subseteq E$. Then

(i) $<F,A,C_F>\tilde{\bigcup}<\varnothing,\varnothing,\varnothing>\cong<F,A,C_F>$

(ii) $<F,A,C_F>\tilde{\bigcap}<\varnothing,\varnothing,\varnothing>\cong<\varnothing,\varnothing,\varnothing>$

(iii) $<F,A,C_F>\tilde{\bigcup}<F,A,C_F>\cong<F,A,C_F>$

(iv) $<F,A,C_F>\tilde{\bigcap}<F,A,C_F>\cong<F,A,C_F>$

(v) $<F,A,C_F>\tilde{\bigcup}<G,B,C_G>$
$\cong<G,B,C_G>\tilde{\bigcup}<F,A,C_F>$

(vi) $<F,A,C_F>\tilde{\bigcap}<G,B,C_G>$
$\cong<G,B,C_G>\tilde{\bigcap}<F,A,C_F>$

(vii) $<F,A,C_F>\tilde{\bigcup}\left(<G,B,C_G>\tilde{\bigcup}<H,C,C_H>\right)$
$\cong\left(<F,A,C_F>\tilde{\bigcup}<G,B,C_G>\right)\tilde{\bigcup}<H,C,C_H>$

$(viii)$ $<F,A,C_F>\tilde{\bigcap}\left(<G,B,C_G>\tilde{\bigcap}<H,C,C_H>\right)$
$\cong\left(<F,A,C_F>\tilde{\bigcap}<G,B,C_G>\right)\tilde{\bigcap}<H,C,C_H>$

(ix) $<F,A,C_F>\tilde{\bigcup}\left(<G,B,C_G>\tilde{\bigcap}<H,C,C_H>\right)$
$\cong\left(<F,A,C_F>\tilde{\bigcup}<G,B,C_G>\right)$
$\tilde{\bigcap}\left(<F,A,C_F>\tilde{\bigcup}<H,C,C_H>\right)$

(x) $<F,A,C_F>\tilde{\bigcap}\left(<G,B,C_G>\tilde{\bigcup}<H,C,C_H>\right)$
$\cong\left(<F,A,C_F>\tilde{\bigcap}<G,B,C_G>\right)$
$\tilde{\bigcup}\left(<F,A,C_F>\tilde{\bigcap}<H,C,C_H>\right)$

International Journal of Innovation in Science and Mathematics
Volume 2, Issue 4, ISSN (Online): 2347–9051

Proof:

(*i*) $< F,A,C_F > \tilde{\bigcup} < \varnothing,\varnothing,\varnothing > \cong < F \cup \varnothing, A \cup \varnothing = A, C_{F\cup\varnothing} >$

We show that $(F \cup \varnothing)(e) = F(e)$

for all $e \in A \cup \varnothing = A$

Now $(F \cup \varnothing)(e) = F(e) \cup \varnothing(e)$

$= F(e) \cup \varnothing = F(e)$ (since $e \notin \varnothing$)

In particular, $F \cup \varnothing = F$ *for all* $e \in A.$

Hence, $< F \cup \varnothing, A \cup \varnothing = A, C_{F\cup\varnothing} > \cong < F,A,C_F >$ and

$< F,A,C_F > \tilde{\bigcup} < \varnothing,\varnothing,\varnothing > \cong < F,A,C_F >$

(*ii*) $< F,A,C_F > \tilde{\bigcap} < \varnothing,\varnothing,\varnothing >$

$\cong < F \cap \varnothing, A \cap \varnothing = \varnothing, C_{F\cap\varnothing} >$

Since $A \cap \varnothing = \varnothing,$ we have

$F \cap \varnothing = \varnothing$ *and* $C_{F\cap\varnothing} = \varnothing$

In particular,

$< F \cap \varnothing, A \cap \varnothing = \varnothing, C_{F\cap\varnothing} > \cong < \varnothing,\varnothing,\varnothing >$ and

$< F,A,C_F > \tilde{\bigcap} < \varnothing,\varnothing,\varnothing > \cong < \varnothing,\varnothing,\varnothing >$

(*iii*) $< F,A,C_F > \tilde{\bigcup} < F,A,C_F >$

$\cong < F \cup F, A \cup A = A, C_{F\cup F} >$

We show that $(F \cup F)(e) = F(e)$ *for all* $e \in A.$

But $(F \cup F)(e) = F(e) \cup F(e) = F(e)$

(by definition and theorem 2.6)

Thus, $F \cup F = F$. In particular,

$< F \cup F, A \cup A = A, C_{F\cup F} > \cong < F,A,C_F >$ and

$< F,A,C_F > \tilde{\bigcup} < F,A,C_F > \cong < F,A,C_F >$

(*iv*) $< F,A,C_F > \tilde{\bigcap} < F,A,C_F >$

$\cong < F \cap F, A \cap A = A, C_{F\cap F} >$

We show that $(F \cap F)(e) = F(e)$ *for all* $e \in A.$

But $(F \cap F)(e) = F(e) \cap F(e) = F(e)$

(by definition and theorem 2.6)

Thus, $F \cap F = F$. In particular,

$< F \cap F, A \cap A = A, C_{F\cap F} > \cong < F,A,C_F >$ and

$< F, A, C_F > \tilde{\bigcap} < F, A, C_F > \cong < F,A,C_F >$

(*v*) $< F,A,C_F > \tilde{\bigcup} < G,B,C_G >$

$\cong < F \cup G, A \cup B = B \cup A, C_{F\cup G} >$ We show that $(F \cup G)(e) = (G \cup F)(e)$ *for all* $e \in A \cup B$. But

$(F \cup G)(e) = F(e) \cup G(e) = G(e) \cup F(e)$

$= (G \cup F)(e)$ (by definition and theorem 2.6)

Thus, $F \cup G = G \cup F$. In particular,

$< F \cup G, A \cup B = B \cup A, C_{F\cup G}$

$> \cong < G \cup F, B \cup A, C_{G\cup F} >$

$= < G,B,C_G > \tilde{\bigcup} < F,A,C_F >$ and

$< F,A,C_F > \tilde{\bigcup} < G,B,C_G >$

$\cong < G,B,C_G > \tilde{\bigcup} < F,A,C_F >$

(*vi*) $< F,A,C_F > \tilde{\bigcap} < G,B,C_G >$

$\cong < F \cap G, A \cap B = B \cap A, C_{F\cap G} >$

We show that $(F \cap G)(e) = (G \cap F)(e)$

for all $e \in A \cap B$.But $(F \cap G)(e)$

$= F(e) \cap G(e) = G(e) \cap F(e)$

$= (G \cap F)(e)$ (by definition and theorem 2.6)

Thus, $F \cap G = G \cap F$. In particular,

$< F \cap G, A \cap B = B \cap A, C_{F\cap G} >$

$\cong < G \cap F, B \cap A, C_{G\cap F} >$

$\cong < G,B,C_G > \tilde{\bigcap} < F,A,C_F >$ *and*

$< F,A,C_F > \tilde{\bigcap} < G,B,C_G >$

$\cong < G,B,C_G > \tilde{\bigcap} < F,A,C_F >$

(vii) $< F,A,C_F > \tilde{\bigcup} \left(< G,B,C_G > \tilde{\bigcup} < H,C,C_H > \right)$

$\cong < F,A,C_F > \tilde{\bigcup} < G \cup H, B \cup C, C_{G\cup H} >$

$\cong < F \cup (G \cup H), A \cup (B \cup C), C_{F\cup(G\cup H)} >$

But $A \cup (B \cup C) = (A \cup B) \cup C$ (1)

We show that $F \cup (G \cup H) = (F \cup G) \cup H$

Now $(F \cup (G \cup H))(e)$

$= F(e) \cup (G \cup H)(e)$ *for all* $e \in A \cup (B \cup C)$

$= F(e) \cup (G(e) \cup H(e))$

$= (F(e) \cup G(e)) \cup H(e) = (F \cup G)(e) \cup H(e)$

$= ((F \cup G) \cup H)(e)$ (by definition and theorem 2.6)

Thus, $F \cup (G \cup H) = (F \cup G) \cup H$ (2)

From (1) and (2), we have,

$< F \cup (G \cup H), A \cup (B \cup C), C_{F\cup(G\cup H)} >$

$\cong < (F \cup G) \cup H, (A \cup B) \cup C, C_{(F\cup G)\cup H} >$

$= < (F \cup G), (A \cup B), C_{F\cup G} > \tilde{\bigcup} < H,C,C_H >$

$\cong \left(< F,A,C_F > \tilde{\bigcup} < G,B,C_G > \right) \tilde{\bigcup} < H,C,C_H >$

i.e $< F,A,C_F > \tilde{\bigcup} \left(< G,B,C_G > \tilde{\bigcup} < H,C,C_H > \right)$

$\cong \left(< F,A,C_F > \tilde{\bigcup} < G,B,C_G > \right) \tilde{\bigcup} < H,C,C_H >$

(*viii*) $< F,A,C_F > \tilde{\bigcap} \left(< G,B,C_G > \tilde{\bigcap} < H,C,C_H > \right)$

$\cong < F,A,C_F > \tilde{\bigcap} < G \cap H, B \cap C, C_{G\cap H} >$

$\cong < F \cap (G \cap H), A \cap (B \cap C), C_{F\cap(G\cap H)} >$

But $A \cap (B \cap C) = (A \cap B) \cap C$ (1)

We show that $F \cap (G \cap H) = (F \cap G) \cap H$

Now $(F \cap (G \cap H))(e) = F(e) \cap (G \cap H)(e)$

for all $e \in A \cap (B \cap C)$

$= F(e) \cap (G(e) \cap H(e)) = (F(e) \cap G(e)) \cap H(e)$

(by definition and theorem 2.6)

$= (F \cap G)(e) \cap H(e) = ((F \cap G) \cap H)(e)$.

Hence, $F \cap (G \cap H) = (F \cap G) \cap H$ (2)

From (1) and (2) we have.

$< F \cap (G \cap H), A \cap (B \cap C), C_{F \cap (G \cap H)} >$

$\tilde{=} < (F \cap G) \cap H, (A \cap B) \cap C, C_{(F \cap G) \cap H} >$

$\tilde{=} < (F \cap G), (A \cap B), C_{(F \cap G)} > \tilde{\cap} < H, C, C_H >$

$= (< F, A, C_F > \tilde{\cap} < G, B, C_G >) \tilde{\cap} < H, C, C_H >$

In particular,

$< F, A, C_F > \tilde{\cap} (< G, B, C_G > \tilde{\cap} < H, C, C_H >)$

$\tilde{=} (< F, A, C_F > \tilde{\cap} < G, B, C_G >) \tilde{\cap} < H, C, C_H >$ (ix)

note that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (1)

Since $C_{F \cup (G \cap H)} = C_{(F \cup G) \cap (F \cup H)}$

(from theorem 3.1),

we have $(F \cup (G \cap H))(e)$

$= ((F \cup G) \cap (F \cup H))(e)$

for all $e \in A \cap (B \cup C)$ (by definition) .

In particular,

$F \cup (G \cap H) = (F \cup G) \cap (F \cup H)$ (2)

Thus, $< F \cup (G \cap H), A \cup (B \cap C), C_{F \cup (G \cap H)} >$

$\tilde{=} < (F \cup G) \cap (F \cup H),$

$(A \cup B) \cap (A \cup C), C_{(F \cup G) \cap (F \cup H)} >$

(from (1) and (2))

$\tilde{=} < F \cup G, A \cup B, C_{F \cup G} >$

$\tilde{\cap} < F \cup H, A \cup C, C_{F \cup H} >$

$\tilde{=} (< F, A, C_F > \tilde{\cup} < G, B, C_G >)$

$\tilde{\cap} (< F, A, C_F > \tilde{\cup} < H, C, C_H >)$

In particular, we have

$< F, A, C_F > \tilde{\cup} (< G, B, C_G > \tilde{\cap} < H, C, C_H >)$

$\tilde{=} (< F, A, C_F > \tilde{\cup} < G, B, C_G >)$

$\tilde{\cap} (< F, A, C_F > \tilde{\cup} < H, C, C_H >)$

(x) $< F, A, C_F > \tilde{\cap} (< G, B, C_G > \tilde{\cup} < H, C, C_H >)$

$\tilde{=} < F, A, C_F > \tilde{\cap} < G \cup H, B \cup C, C_{G \cup H} >$

$\tilde{=} < F \cap (G \cup H), A \cap (B \cup C), C_{F \cap (G \cup H)} >$

But $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (1)

We show that $F \cap (G \cup H) = (F \cap G) \cup (F \cap H)$

Now $C_{F \cap (G \cup H)} = C_{(F \cap G) \cup (F \cap H)}$ (from theorem 3.1)

Thus, $(F \cap (G \cup H))(e) = ((F \cap G) \cup (F \cap H))(e)$

for all $e \in A \cap (B \cup C)$ (by definition)

In particular, $F \cap (G \cup H)$

$= (F \cap G) \cup (F \cap H)$ (2)

From (1) and (2) we have,

$< F \cap (G \cup H), A \cap (B \cup C), C_{F \cap (G \cup H)} >$

$\tilde{=} < (F \cap G) \cup (F \cap H),$

$(A \cap B) \cup (A \cap C), C_{(F \cap G) \cup (F \cap H)} >$

$\tilde{=} < F \cap G, A \cap B, C_{F \cap G} > \tilde{\cup} < F \cap H, A \cap C, C_{F \cap H} >$

$\tilde{=} (< F, A, C_F > \tilde{\cap} < G, B, C_G >)$

$\tilde{\cup} (< F, A, C_F > \tilde{\cap} < H, C, C_H >)$

In particular, we have

$< F, A, C_F > \tilde{\cap} (< G, B, C_G > \tilde{\cup} < H, C, C_H >)$

$\tilde{=} (< F, A, C_F > \tilde{\cap} < G, B, C_G >)$

$\tilde{\cup} (< F, A, C_F > \tilde{\cap} < H, C, C_H >)$

Corollary 3.9.

Let $< F, A, C_F >$ and $< G, B, C_G >$ *be a soft msets* over the universal mset $U \in M_n^t(X)$ and E be a set of parameters such that $A, B, C \subseteq E$. Then

(i) $< F, A, C_F > \tilde{\cup} (< F, A, C_F > \tilde{\cap} < G, B, C_G >) \tilde{=} < F, A, C_F >$

(ii) $< F, A, C_F > \tilde{\cap} (< F, A, C_F > \tilde{\cup} < G, B, C_G >) \tilde{=} < F, A, C_F >$

(absorbtion laws)

Proof:

(i) $< F, A, C_F > \tilde{\cup} (< F, A, C_F > \tilde{\cap} < G, B, C_G >)$

$\tilde{=} < F, A, C_F > \tilde{\cup} < F \cap G, A \cap B, C_{F \cap G} >$

$\tilde{=} < F \cup (F \cap G), A \cup (A \cap B), C_{F \cup (F \cap G)} >$

But $A \cup (A \cap B) = A$ (1)

We show that

$F \cup (F \cap G) = F$. Now $(F \cup (F \cap G))(e)$

$= F(e) \cup (F \cap G)(e)$ for all $e \in A \cup (A \cap B)$

In particular,

$(F \cup (F \cap G))(e) = F(e) \cup (F(e) \cap G(e))$

for all $e \in A \cup (A \cap B)$

But $F(e) \cup (F(e) \cap G(e)) = F(e)$

(absorbtion law for msets)

Thus, $(F \cup (F \cap G))(e) = F(e)$ and

$F \cup (F \cap G) = F$ (2)

Now from (1) and (2) we have,

$< F \cup (F \cap G), A \cup (A \cap B), C_{F \cup (F \cap G)} > \tilde{=} < F, A, C_F >$

In particular, we have:

$< F, A, C_F > \tilde{\cup} (< F, A, C_F > \tilde{\cap} < G, B, C_G >) \tilde{=} < F, A, C_F >$

(ii) $<F,A,C_F>\tilde{\cap}(<F,A,C_F>\tilde{\cup}<G,B,C_G>)$

$\cong(<F,A,C_F>\tilde{\cap}<F,A,C_F>)$

$\tilde{\cup}(<F,A,C_F>\tilde{\cap}<G,B,C_B>)$

(from proposition 3.8 (x))

$\cong<F,A,C_F>\tilde{\cup}(<F,A,C_F>\tilde{\cap}<G,B,C_B>)$

(from proposition 3.8 (iv))

$\cong<F,A,C_F>$ (from (i) above)

Proposition 3.10. *Let* $<F,A,C_F>$ *and* $<G,B,C_G>$ be a soft msets over the universal mset $U\in M_n^t(X)$ and E be a set of parameters such that $A,B\subseteq E$.

Then

(i) $((<F,A,C_F>)^c)^c\cong<F,A,C_F>$

(ii) $(<F,A,C_F>\tilde{\cup}<G,B,C_G>)^c$

$\cong(<F,A,C_F>)^c\tilde{\cap}(<G,B,C_G>)^c$

(iii) $(<F,A,C_F>\tilde{\cap}<G,B,C_G>)^c$

$\cong(<F,A,C_F>)^c\tilde{\cup}(<G,B,C_G>)^c$

Proof:

(i) $((<F,A,C_F>)^c)^c$

$\cong(<F^c,A',C_{F^c}>)^c\cong<(F^c)^c,(A')',C_{(F^c)^c}>$

$\cong<(F^c)^c,A,C_{(F^c)^c}>$ (since $(A')'=A$)

We show that $(F^c)^c=F$

Now $(F^c)^c(e)=(\mu-F^c)(e)$

$=\mu(e)-F^c(e)=\mu(e)-(\mu(e)-F(e))$

$=F(e)$ (see theorem 2.4) for all $e\in(A')'=A$(ii)

Thus, $(F^c)^c=F$. In particular,

$((<F,A,C_F>)^c)^c$

$\cong<(F^c)^c,A,C_{(F^c)^c}>$

$\cong<F,A,C_F>$

$(<F,A,C_F>\tilde{\cup}<G,B,C_G>)^c$

$\cong(<F\cup G,A\cup B,C_{F\cup G}>)^c$

$=<(F\cup G)^c,(A\cup B)',C_{(F\cup G)^c}>$

$\cong<(F\cup G)^c,A'\cap B',C_{(F\cup G)^c}>$

such that $(F\cup G)^c(e)=U$

for all $e\in(A\cup B)'=A'\cap B'$

We show that $(F\cup G)^c=F^c\cap G^c$.

Now for all $e\in A'\cap B'$, we have

$F^c(e)=U$ and $G^c(e)=U$

$\therefore$ $F^c(e)\cap G^c(e)=U$ *for all* $e\in A'\cap B'$

In particular, $(F^c\cap G^c)(e)=U$

for all $e\in(A\cup B)'$

Thus, $(F\cup G)^c=F^c\cap G^c$ and

$<(F\cup G)^c,A'\cap B',C_{(F\cup G)^c}>$

$\cong<F^c\cap G^c,A'\cap B',C_{F^c\cap G^c}>$

i.e $<(F\cup G)^c,(A\cup B)',C_{(F\cup G)^c}>$ (iii)

$\cong<F^c\cap G^c,A'\cap B',C_{F^c\cap G^c}>$

$\cong<F^c,A',C_{F^c}>\cap<G^c,B',C_{G^c}>$

$\cong(<F,A,C_F>)^c\tilde{\cap}(<G,B,C_G>)^c$

In particular, $(<F,A,C_F>\tilde{\cup}<G,B,C_G>)^c$

$\cong(<F,A,C_F>)^c\tilde{\cap}(<G,B,C_G>)^c$

$(<F,A,C_F>\tilde{\cap}<G,B,C_G>)^c$

$\cong(<F\cap G,A\cap B,C_{F\cap G}>)^c$

$\cong<(F\cap G)^c,(A\cap B)',C_{(F\cap G)^c}>$

such that $(F\cap G)^c(e)=U$.

We show that $(F\cap G)^c=F^c\cup G^c$

Now $F^c(e)=U$ and $G^c(e)=U$

for all $e\in A'$ and all $e\in B'$ respectively.

In particular, $F^c(e)\cup G^c(e)=(F^c\cup G^c)(e)=U$

for all $e\in A'\cup B'$. But $A'\cup B'=(A\cap B)'$

(Demorgan's law for sets).

Thus $(F^c\cup G^c)(e)=U$ *for all* $e\in(A\cap B)'$

In particular, $(F\cap G)^c=F^c\cup G^c$

Hence, $<(F\cap G)^c,(A\cap B)',C_{(F\cap G)^c}>$

$\cong<F^c\cup G^c,A'\cup B',C_{F^c\cup G^c}>$

$\cong<F^c,A',C_{F^c}>\tilde{\cup}<G^c,B',C_{G^c}>$

$\cong(<F,A,C_F>)^c\tilde{\cup}(<G,B,C_G>)^c$

$\therefore(<F,A,C_F>\tilde{\cap}<G,B,C_G>)^c$

$\cong(<F,A,C_F>)^c\tilde{\cup}(<G,B,C_G>)^c$

Proposition 3.11. Let $<F,A,C_F>$ be a soft mset over the universal mset $U\in M_n^t(X)$ and E be a set of parameters such that $A\subseteq E$. Then $(<F,A,C_F>)^c\tilde{\cap}<F,A,C_F>\cong<\varnothing,\varnothing,\varnothing>$

Proof:

$(< F,A,C_F >)^c \tilde{\bigcap} < F,A,C_F >$

$\tilde{=} < F^c,A',C_{F^c} > \tilde{\bigcap} < F,A,C_F >$

$\tilde{=} < F^c \bigcap F, A' \bigcap A, C_{F^c \cap F} >$.

We show that $F^c \bigcap F = \varnothing$

and $C_{F^c \cap F} = \varnothing$.

Now $A' \bigcap A = \varnothing$.

Hence, $F^c \bigcap F = \varnothing$ *and* $C_{F^c \cap F} = \varnothing$.

In particular,

$< F^c \bigcap F, A' \bigcap A, C_{F^c \cap F} > \tilde{=} < \varnothing,\varnothing,\varnothing >$ and

$(< F,A,C_F >)^c \tilde{\bigcap} < F,A,C_F > = < \varnothing,\varnothing,\varnothing >$

But in general,

$(< F,A,C_F >)^c \tilde{\bigcup} < F,A,C_F > \tilde{\neq} < \mu,E,C_\mu >$

Since $(F^c \bigcup F)(e) \neq U$ *for all* $e \in A' \bigcup A = E$

However if $< F,A,C_F >$ is a whole soft mset , then we have

$(< F,A,C_F >)^c \tilde{\bigcup} < F,A,C_F > \tilde{=} < \mu,E,C_\mu >$

Since $(F^c \bigcup F)(e) = U$ *for all* $e \in A' \bigcup A = E$

IV. Conclusion

In this paper, the concept of soft multiset has been redefined with multisets as integral part. A universal soft multiset also defined leading to a redefinition of complement of a soft multisets.

From the new perspective, some of the algebraic properties such as commutativity, associativity, distributivity absorbtion, idempotent and demorgan's laws are valid.

Though in general under this concept, the axiom of contradiction is satisfied with the exception of exclusion, the condition under which both are satisfied has been established.

Acknowledgement

The author acknowledges the constructive editorial comments of this article and the acceptance to disseminate its contents to the wider community for accessibility. In particular, my appreciation goes to the reviewers for their time devoted toward having an acceptable paper of this quality.

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