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Path Related Product Cordial Graph

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***Abstract* – Let G = (V,E) be a graph with p vertices and q edges. A Product cordial labeling of a Graph G with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label (f(u).f(v)) with the condition the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a product cordial labeling is called a product cordial graph (PCG). In this paper, we proved that path related graphs Path P_n , Fan P_n+K_1 , $[P_n : S_m]$: (n - even) , comb $P_n \odot K_1$, $[P_n : C_m]$: (n-even) are product cordial graphs.**

***Keywords* – Fan, Comb, Join of Two Graphs, Product Cordial Labeling, Product Cordial Graph.**

2000 Mathematics Subject Classification 05C78.

I. Introduction

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {uv} of vertices in E is called an edge or a line of G. In this paper , we proved that path related graphs Path P_n, Fan P_n+K_1 , $[P_n : S_m]$: (n - even) , comb $P_n \odot K_1$ are product cordial graphs. For graph theory terminology, we follow [2].

II. Preliminaries

Let G = (V,E) be a graph with p vertices and q edges. A Product cordial labeling of a Graph G [1,4,5] with vertex set V is a bijection from V to {0,1} such that if each edge uv is assigned the label (f(u) . f(v)) with the condition the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a product cordial labeling is called a product cordial graph (PCG). In this paper, we proved that path related graphs Path P_n , Fan P_n+K_1 , $[P_n : S_m]$: (n - even), comb $P_n \odot K_1$ are product cordial graphs.

Definition: 2.1 [3]

P_n is a path of length n-1.

Definition: 2.2 [3]

The join $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining v_1 and v_2 as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V (G_2)$ and edge set $E (G_1 \cup G_2) = E (G_1) \cup E (G_2) \cup [$ uv: $u \in V(G_1)$ and $v \in V(G_2)$ .The graph $P_n + K_1$ is called a Fan.

Definition: 2.3 [3]

$[P_n : S_m]$ is a graph obtained from a path P_n by joining every vertex of a path to a root of a star S_m by an edge.

Definition: 2.4 [3]

The corona $G_1 \odot G_2$ of two Graphs G_1 and G_2 is defined as the Graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2. The graph $P_n \odot K_1$ is called a comb.

III. Path Related Product Cordial Graph

Theorem: 3.1

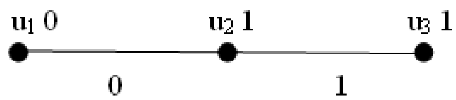
Path P_n : (n−odd) is product cordial.

Proof:

Let G be P_n .
Let V (G) = { $[u_i : 1 \le i \le n]$ } and
E(G) = { [$(u_iu_{i+1}) : 1 \le i \le n-1$] }.
Define $f : V(G) \to \{0,1\}$.

Case: 1

When n =3,
The labeling is,



Case: 2

When n>3,
The vertex labelings are,

$f(u_1) = 0$
$f(u_i) = 1 \quad 2 \le i \le (n+3)/2$
$f(u_i) = 0 \quad (n+5)/2 \le i \le n.$

The induced edge labelings are,

$f^*(u_1u_2) = 0$
$f^*(u_iu_{i+1}) = 1 \quad 2 \le i \le (n+1)/2$
$f^*(u_iu_{i+1}) = 0 \quad (n+3)/2 \le i \le n.$

Here $v_f(0)+1 = v_f(1)$ for all n and $e_f(0) = e_f(1)$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $|e_f(0) - e_f(1) | \le 1$.

Hence, P_n : (n-odd) is product cordial.

For example, the product cordial labeling of P_5 is shown in figure3.2.

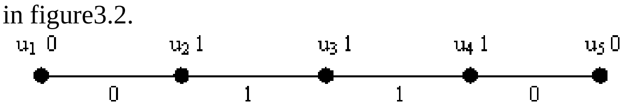


Fig.3.2 : P_5

Theorem: 3.3

Path P_n : (n−even) is product cordial.

Proof:

Let V (P_n) = { $[u_i : 1 \le i \le n]$ } and
E (P_n) = { [$(u_iu_{i+1}) : 1 \le i \le n-1$] }.
Define $f : V(G) \to \{0,1\}$.

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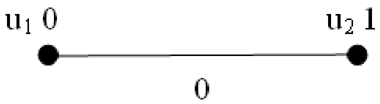
Case: 1

When n =2,

The labeling is,



Case: 2

When n>2,

The vertex labelings are,

$f(u_1) = 0$.

$f(u_i) = 1 \quad 2 \le i \le (n+2)/2$

$f(u_i) = 0 \quad (n+4)/2 \le i \le n$.

The induced edge labelings are,

$f^*(u_1u_2) = 0$.

$f^*(u_iu_{i+1}) = 1 \quad 2 \le i \le n/2$

$f^*(u_iu_{i+1}) = 0 \quad (n+2)/2 \le i \le n$.

Here $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1) + 1$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, P_n : (n-even) is product cordial.

For example the product cordial labeling of P_6 is shown in figure 3.4.

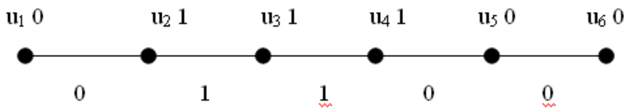


Fig.3.4 P_6

Theorem: 3.5

Fan P_n+K_1 : (n-even) is product cordial.

Proof:

Let G be P_n+K_1.

Let V(G) = { [u, u_i : $1 \le i \le n$] } and

E(G) = { [($u u_i$) : $1 \le i \le n$] U [(u_iu_{i+1}) : $1 \le i \le n-1$] }.

Define $f: V(G) \to \{0,1\}$

The vertex labelings are ,

$f(u) = 1$

$f(u_i) = 0 \quad 1 \le i \le n/2$

$f(u_i) = 1 \quad (n+2)/2 \le i \le n$

The induced edge labelings are,

$f^*[(u u_i)] = 0 \quad 1 \le i \le n/2$

$f^*[(u u_i)] = 1 \quad (n+2)/2 \le i \le n$.

Here, $v_f(1) = v_f(0) + 1$ for all n and

$e_f(0) = e_f(1) + 1$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, P_n+K_1 : (n-even) is product cordial.

For example the product cordial labeling of P_4+K_1 is shown in the figure 3.6.

Theorem: 3.7

Graph P_n+K_1 : (n-odd) is not product cordial.

Proof:

Let G be P_n+K_1 : (n-odd).

Let V(G) = { [u, u_i : $1 \le i \le n$] } and

E(G) = { [(u, u_i) : $1 \le i \le n$] U [(u_iu_{i+1}) : $1 \le i \le n-1$] }

If f (u) = 0, then for all n edges $f^*[(u u_i)] = 0$, $1 \le i \le n$ and even if at least one u_i assigned label 0 it does not satisfy $|e_0(f) - e_1(f)| \le 1$.

If f(u) = 1 and [(n+1)/2] vertices are assigned 0 label it does not satisfy $|e_f(0) - e_f(1)| \le 1$.

Otherwise, it violates the condition $| v_f(0) - v_f(1) | \le 1$.

Hence P_n+K_1 : (n-odd) is not product cordial.

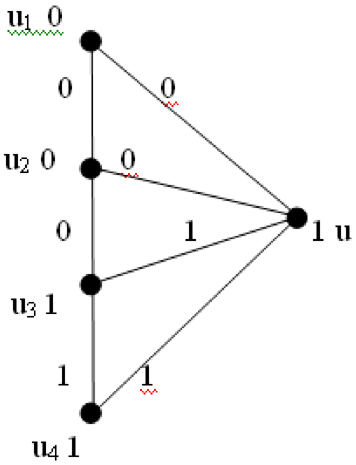


Fig.3.6. $P_4 + K_1$

Theorem: 3.8

Graph $[P_n:S_m]$: (n−even) is product cordial.

Proof:

Let G be $[P_n:S_m]$.

Let V (G) = { [u_i, v_i : $1 \le i \le n$] , [v_{ij} : $1 \le i \le n$, $1 \le j \le m$] and

E(G) = { [(u_iu_{i+1}) : $1 \le i \le n-1$] U [(u_iv_i) : $1 \le i \le n$]
U [(v_iv_{ij}) : $1 \le i \le n$, $1 \le j \le m$] }.

Define $f: V(G) \to \{0,1\}$.

The vertex labelings are,

$f(u_1) = f(v_1) = 0$.

$f(u_i) = f(v_i) = 1 \quad 2 \le i \le (n+2)/2$.

$f(u_i) = f(v_i) = 0 \quad (n+4)/2 \le i \le n$.

$f(v_{1j}) = 0 \quad 1 \le j \le m$.

$f(v_{ij}) = 1 \quad 2 \le i \le (n+2)/2$, $1 \le j \le m$.

$f(v_{ij}) = 0 \quad (n+4)/2 \le i \le n$, $1 \le j \le m$.

The induced edge labelings are,

$f^*(u_1u_2) = 0$.

$f^*(u_iu_{i+1}) = 1 \quad 2 \le i \le n/2$.

$f^*(u_iu_{i+1}) = 0 \quad (n+2)/2 \le i \le n-1$.

$f^*(u_1v_1) = 0$

$f^*(u_iv_i) = 1 \quad 2 \le i \le (n+2)/2$.

$f^*(u_iv_i) = 0 \quad (n+4)/2 \le i \le n$.

$f^*(v_1v_{1j}) = 0 \quad 1 \le j \le m$.

$f^*(v_iv_{ij}) = 1 \quad 2 \le i \le (n+2)/2$, $1 \le j \le m$.

$f^*(v_iv_{ij}) = 0 \quad (n+4)/2 \le i \le n$, $1 \le j \le m$.

Here $v_f(0) = v_f(1)$ for all n and $e_f(1) + 1 = e_f(0)$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, $[P_n:S_m]$: (n-even) is product cordial.

For example, the product cordial labeling of $[P_4:S_2]$ is shown in figure 3.9.

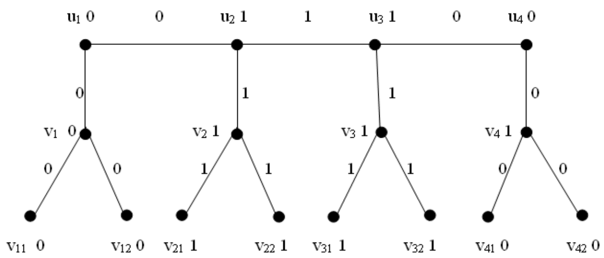


Fig.3.9. $[P_4:S_2]$

Theorem: 3.10

Comb $P_n \odot K_1$ is product cordial.

Proof:

Let G be $P_n \odot K_1$.

Let V (G) = $\{ [u_i, v_i : 1 \leq i \leq n] \}$ and

E(G) = $\{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \}$.

Define $f : V(G) \rightarrow \{0,1\}$.

The vertex labelings are,

$f(u_i) = 1 \quad 1 \leq i \leq n$

$f(v_i) = 0 \quad 1 \leq i \leq n$

The induced edge labelings are,

$f^*[(u_i u_{i+1})] = 1 \quad 1 \leq i \leq n-1$

$f^*[(u_i v_i)] = 0 \quad 1 \leq i \leq n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(1) + 1 = e_f(0)$ for all n.

Therefore, the graph G satisfies the conditions $| v_f(0) - v_f(1) | \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $P_n \odot K_1$ is product cordial.

For example, the product cordial labeling of $P_4 \odot K_1$ is shown in figure 3.11.

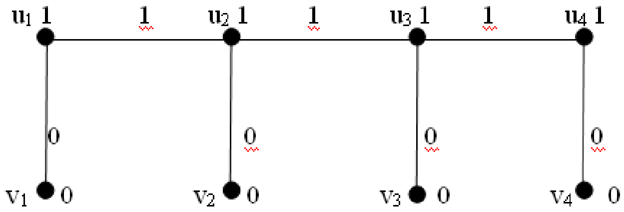


Fig.3.11. $P_4 \odot K_1$

IV. Conclusion

Graph labeling has many practical applications, particularly cordial labeling and product cordial labeling play a vital role in digital technology. In this paper product cordial labeling of some of the graphs are identified.

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