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Some Special Class of Odd-Even Graceful Graphs

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***Abstract* – The Odd-Even graceful labeling of a graph G with q edges means that there is an injection f : V(G) to {1,3,5,…,2q+1} such that, when each edge uv is assigned the label $|f(u)-f(v)|$, the resulting edge labels are {2,4,6,…,2q }.A graph which admits an odd-even graceful labeling is called an odd-even graceful graph. In this paper we prove that some graphs namely SP $(P_n, K_{1,m})$, $S_{m,n}$ $(n: odd)$, $S_{m,n}$ $(n: even)$, C_4 @ S_m , $P_{(n)_m}(n: odd)$, $P_{(n)_m}$ $(n: even)$ are Odd-Even graceful.**

***Keywords and Phrases*: Graceful Labeling, Odd Graceful Labeling, Odd-Even Graceful Graph.**
AMS 2000 Mathematics subject classification: 05C78

I. Introduction

As a graph, we mean a finite undirected graph without loops or multiple edges. A path on n vertices is denoted as P_n . G^+ is a graph obtained from the graph G as attaching a pendant vertex at each vertex of G. Graph labeling, where the vertices are assigned certain values subject to some conditions, have often motivated as practical problems. In the last five decades enormous work has been done on this subject[1]. The concept of graceful labeling was introduced as Rosa[3] and in 1967.

A function f is a graceful labeling of a graph G with q edges if f is an injection from the vertices of G to the set {0,1,2,…,q} such that, when each edge uv is assigned the label $|f(u)-f(v)|$, the resulting edge labels are distinct. Golomb[2] subsequently called such labeling are graceful and this is now the popular term. A graph G with q edges to be odd-graceful if there is an injection f from V(G) to {0,1,2,…,2q−1} such that, when each edge uv is assigned the label $|f(u)-f(v)|$,the resulting edge labels are {1,3,…,2q−1}. Sridevi R, NavaneethaKrishnan S defined the odd-even graceful labeling[4,5]. *The Odd-Even graceful labeling of a graph G with q edges is an injection f : V(G) to {1,3,5,…,2q+1} such that, when each edge uv is assigned the label* $|f(u)-f(v)|$, *the resulting edge labels are {2,4,6,…,2q }. A graph which admits an odd-even graceful labeling is called an odd-even graceful graph.* In this paper, we prove that some graphs namely SP(P_n , $K_{1,m}$) , $S_{m,n}$ $(n: odd)$, $S_{m,n}$ $(n: even)$, C_4 @ S_m , $P_{(n)_m}$ $(n: odd)$, $P_{(n)_m}$ $(n: even)$ are Odd-Even graceful.

II. Main Results

In this section we prove some graphs namely SP(P_n , $K_{1,m}$) , $S_{m,n}$ $(n: odd)$ $S_{m,n}$ $(n: even)$, C_4 @ S_m , $P_{(n)_m}$ $(n: odd)$, $P_{(n)_m}$ $(n: even)$ are Odd-Even graceful.

Definition 2.1. A (p,q)-graph G = (V,E) is said to be an Odd-Even graceful if there exists an injection f from V into { 1,3,5,…,2q+1} such that f*(E)= {2,4,6,…,2q } where f*(uv) = |f(u) − f(v)| for any edge uvϵE.

Definition 2.2. A graph obtained from a path of length $n-1$ by attaching root of a star $K_{1,m}$ at one end of the path. It is denoted by SP(P_n , $K_{1,m}$).

Theorem 2.3. SP(P_n , $K_{1,m}$) is an Odd-Even graceful graph.

Proof: Let G = SP(P_n , $K_{1,m}$)

Let V(G) = $\{(v_i : 1 \leq i \leq n), (u_i : 1 \leq i \leq m)\}$

Let E(G) = $\{(v_i v_{i+1}) : 1 \leq i \leq n-1\} \cup \{(v_n u_i) : 1 \leq i \leq m\}$ and

$|V(G)|$ = m+n and $|E(G)|$ = m+n −1 .

Case (i): when n is odd .

Define f: V(G) → {1,3,5,…,(2n+2m−1)} as

f(v_{2i-1}) = 2i−1, $1 \leq i \leq \frac{n+1}{2}$

f(v_{2i})=$(2m+1)+2(n-1)-2(i-1), 1 \leq i \leq \frac{n-1}{2}$

f(u_i) = n+$2i, 1 \leq i \leq m.$

The induced edge labeling are given as

f*($v_i v_{i+1}$)=2m+2(n−1)−2(i−1) , $1 \leq i \leq n-1$

f*($v_n u_i$) = 2i , $1 \leq i \leq m.$

It shows the edge labels are distinct.

Hence, SP(P_n , $K_{1,m}$) is an Odd-Even graceful graph.

For example the Odd-Even graceful labeling of SP(P_5 , $K_{1,6}$) is shown in *figure 1.*

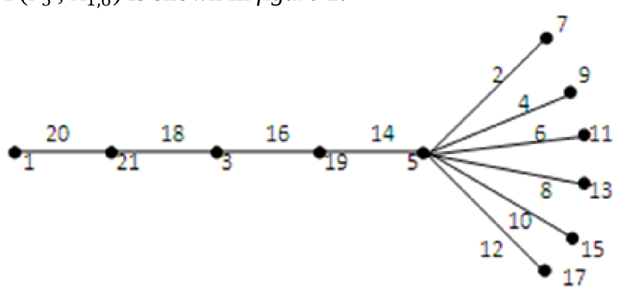

Fig.1. *SP*(P_5, $K_{1,6}$)

Case (ii): when n is even

Define f: V(G) → {1,3,5,…,(2n+2m−1)} as

f(v_{2i-1}) = 2i−1, $1 \leq i \leq \frac{n}{2}$

f(v_{2i})=$(2m+1)+2(n-1)-2(i-1), 1 \leq i \leq \frac{n}{2}$

f(u_i) = n$-1+2i, 1 \leq i \leq m.$

The induced edge labeling are given as

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$f^*(v_i\, v_{i+1}) = 2m+2(n-1)-2(i-1)\,,\ 1 \le i \le n-1$
$f^*(v_n\, u_i) = 2i\,,\ 1 \le i \le m.$
It shows the edge labels are distinct.
Hence, SP(P_n , $K_{1,m}$) is an Odd-Even graceful graph.
For example the Odd-Even graceful labeling of SP(P_6, $K_{1,7}$) is shown in *figure 2*.

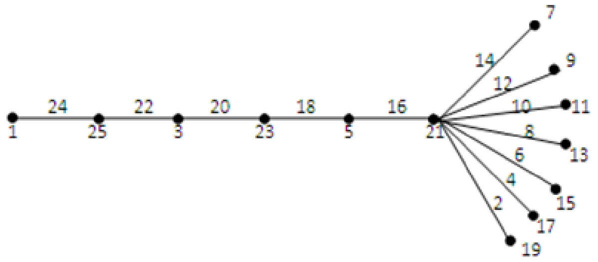

Fig.2. *SP*(P_6, $K_{1,7}$)

Definition 2.6. Graph $S_{m,n}$ $(n > 2)$ is a star with n spokes in which each spoke is a path of length m.

Theorem 2.7. $S_{m,n}$ $(n{:}\,odd)$ is an Odd-Even graceful graph.

Proof: Let G = $S_{m,n}$
Let V(G) = $\{u\,, u_{ij} : 1 \le i \le m, 1 \le j \le n\}$.
Let E(G) = $\{(uu_{i1} : 1 \le i \le m)\} \cup \{(u_{ij}\, u_{ij+1} : 1 \le i \le m, 1 \le j \le n-1)\}$ and
$|V(G)| = mn + 1$ and $|E(G)| = mn$.
Define f: V(G) → {1,3,5,…,(2nm+1)} as
Let f(u) = 1
When $i \equiv 1\ mod 2$
$f(u_{i\,2j}) = n(i-1) + 2j + 1,\ 1 \le j \le \frac{n-1}{2}\,,\ 1 \le i \le m$
$f(u_{i\,2j-1}) = (2mn+1) - n(i-1) - 2(j-1),\ 1 \le j \le \frac{n+1}{2},\ 1 \le i \le m$.
When $i \equiv 0\ mod 2$
$f(u_{i\,2j-1}) = ni+1 - 2(j-1),\ 1 \le j \le \frac{n+1}{2},\ 1 \le i \le m$
$f(u_{i\,2j}) = 2mn + 3 - ni + 2(j-1),\ 1 \le j \le \frac{n-1}{2},\ 1 \le i \le m$.
The induced edge labeling are given as

$$f^*(uu_{i1}) = \begin{cases} ni\,,\ 1 \le i \le m\,,\ i \equiv 0\ mod 2 \\ 2mn-(i-1)n\,,\ 1 \le i \le m\,,\ i \equiv 1\ mod 2 \end{cases}$$

Case (i): when m is odd .
When $1 \le i \le m\,, 1 \le j \le n-1$

$$f^*(u_{m+1-i,n+1-j}\ u_{m+1-i,n-j}) = \begin{cases} 2j+2n(i-1)\ ,\ i \equiv 1\ mod 2 \\ 2ni-2j\ ,\ i \equiv 0\ mod 2 \end{cases}$$

Case (ii): when m is even .
When $1 \le i \le m\,, 1 \le j \le n-1$

$$f^*(u_{m+1-i,n+1-j}\ u_{m+1-i,n-j}) = \begin{cases} 2ni-2-2(j-i)\ ,\ i \equiv 1\ mod 2 \\ 2n(i-1) + 2j\ ,\ i \equiv 0\ mod 2 \end{cases}$$

It shows the edge labels are distinct.
Hence, $S_{m,n}$ is an Odd-Even graceful graph.
For example the Odd-Even graceful labeling of $S_{5,5}$ and $S_{4,7}$ are shown in figure 2.8. and *figure 3.* respectively

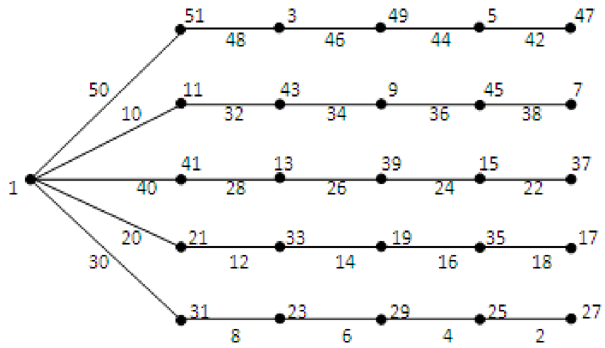

Fig.3. $S_{5,5}$

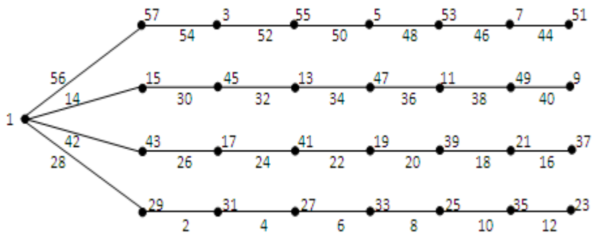

Fig.4. $S_{4,7}$

Theorem 2.10. $S_{m,n}$ $(n{:}\,even)$ is an Odd-Even graceful graph.

Proof: Let G = $S_{m,n}$
Let V(G) = $\{(u), (u_{ij} : 1 \le i \le m, 1 \le j \le n)\}$.
Let E(G) = $\{(uu_{i1} : 1 \le i \le m)\} \cup \{(u_{ij}\, u_{ij+1} : 1 \le i \le m, 1 \le j \le n-1)\}$ and $|V(G)| = mn + 1$ and $|E(G)| = mn$.
Define f: V(G) → {1,3,5,…,(2nm+1)} as
Let f(u) = 1
When $i \equiv 1\ mod 2$
$f(u_{i\,2j}) = n(i-1) + 2j + 1,\ 1 \le j \le \frac{n}{2}\,,\ 1 \le i \le m$
$f(u_{i\,2j-1}) = (2mn+1) - n(i-1) - 2(j-1),\ 1 \le j \le \frac{n}{2},\ 1 \le i \le m$.
When $i \equiv 0\ mod 2$
$f(u_{i\,2j-1}) = ni+1 - 2(j-1),\ 1 \le j \le \frac{n}{2},\ 1 \le i \le m$
$f(u_{i\,2j}) = 2mn + 3 - ni + 2(j-1),\ 1 \le j \le \frac{n}{2},\ 1 \le i \le m$
The induced edge labeling are given as

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$f^*(uu_{i1}) = \begin{cases} ni\,,\ 1 \le i \le m\,,\ i \equiv 0\ mod2 \\ 2mn-(i-1)n\,,\ 1 \le i \le m\,,\ i \equiv 1\ mod2 \end{cases}$

Case (i): when m is odd .

When $1 \le i \le m\,, 1 \le j \le n-1$

$f^*(u_{m+1-i,n+1-j}\ u_{m+1-i,n-j}) = \begin{cases} 2j+2n(i-1)\,,\ i \equiv 1\ mod2 \\ 2ni-2j\,,\ i \equiv 0\ mod2 \end{cases}$

Case (ii): when m is even .

When $1 \le i \le m\,, 1 \le j \le n-1$

$f^*(u_{m+1-i,n+1-j}\ u_{m+1-i,n-j}) = \begin{cases} 2ni-2-2(j-i)\,,\ i \equiv 1\ mod2 \\ 2n(i-1)+2j\,,\ i \equiv 0\ mod2 \end{cases}$

It shows the edge labels are distinct.

Hence, $S_{m,n}$ is an Odd-Even graceful graph.

For example the Odd-Even graceful labeling of $S_{3,6}$ and $S_{6,4}$ are shown in *figure 5 and figure 6.* respectively

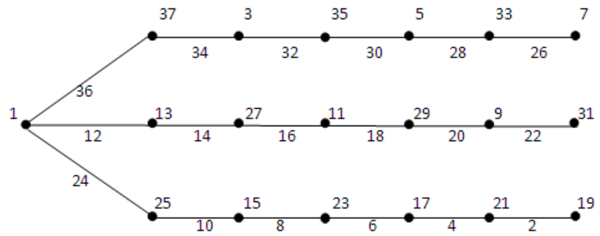

Fig.5. $S_{3,6}$

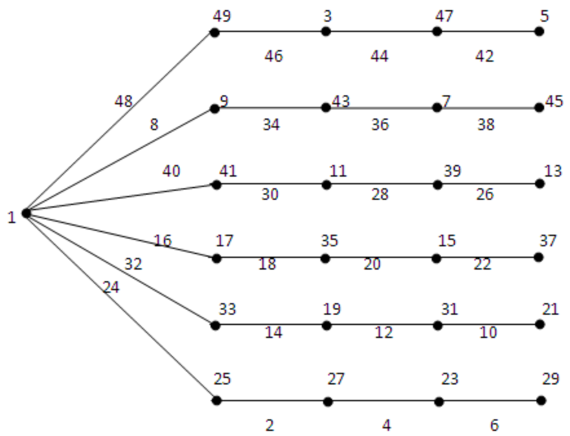

Fig.6. $S_{6,4}$

Definition 2.13. Graph obtained by attaching centre vertex of a star S_m *at* any one of the vertex of C_n is denoted by $C_n @ S_m$.

Theorem 2.14. $C_4 @ S_m$. is an Odd-Even graceful graph.

Proof: Let G = $C_4 @ S_m$.

Let V(G) = $\{(u_i : 1 \le i \le 4), (u_{1j} : 1 \le j \le m)\}$

Let E(G) = $\{(u_1u_3) \cup (u_1u_4) \cup (u_2u_3) \cup (u_2u_4)\} \cup \{(u_1 u_{1j} : 1 \le j \le m)\}$ and

$|V(G)|$ = 4+m and $|E(G)|$ = 4+m .

Define f: V(G) → {1,3,5,…,(2m+9)} as

$f(u_i) = 2i-1, 1 \le i \le 3$

$f(u_4) = 9$

$f(u_{1j}) = 2j+9, 1 \le j \le m.$

The induced edge labeling are given as

$f^*(u_1u_3) = 4$, $f^*(u_1u_4) = 8$,

$f^*(u_2u_3) = 2$, $f^*(u_2u_4) = 6$,

$f^*(u_1 u_{1j}) = 2j+8\ , 1 \le j \le m.$

It shows the edge labels are distinct.

Hence, $C_4 @ S_m$ .is an Odd-Even graceful graph.

For example the Odd-Even graceful labeling of $C_4 @ S_9$. is shown in *figure 7.*

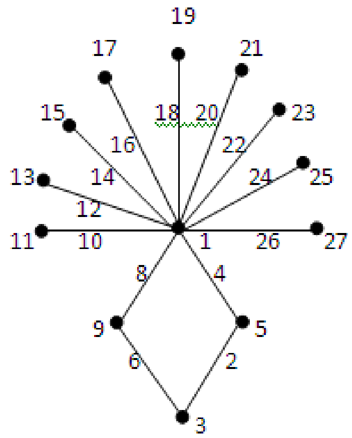

Fig.7. $S_4 @ S_9$

Definition 2.16. Graph $P_{(n)_m}$ = G(V,E) suchthat V(G) = $\{(u_{ij} : 1 \le i \le m, 1 \le j \le n)\}$ and

E(G) = $\{(u_{ij}\, u_{ij+1} : 1 \le i \le m, 1 \le j \le n) \cup [(u_{ij}\, u_{i+1\,j} : 1 \le i \le m-1) for\ any\ fixed\ j]\}$.

Theorem 2.17. $P_{(n)_m}$ $(n: odd)$, is an Odd-Even graceful graph.

Proof: Let G = $P_{(n)_m}$

Let V(G) = $\{(u_{ij} : 1 \le i \le m, 1 \le j \le n)\}$.

Let E(G) = $\{(u_{ij}\, u_{ij+1} : 1 \le i \le m, 1 \le j \le n-1) \cup \left(\left(u_{i\,\frac{n+1}{2}}\ u_{i+1\,\frac{n+1}{2}}\right) : 1 \le i \le m-1\right)\}$ and

$|V(G)|$ = mn and $|E(G)|$ = mn − 1 .

Define f: V(G) → {1,3,5,…,(2nm−1)} as

When $i \equiv 0\ mod2$

$f(u_{i\,2j}) = n(i-2) + 2(j-1) + n + 2, 1 \le j \le \frac{n-1}{2}, 1 \le i \le m$

$f(u_{i\,2j-1}) = (2mn-n) - n(i-2) - 2(j-1), 1 \le j \le \frac{n+1}{2}, 1 \le i \le m$.

When $i \equiv 1\, mod 2$

$f(u_{i\,2j-1}) = n(i-1)+1+2(j-1), 1 \le j \le \frac{n+1}{2}$, $1 \le i \le m$

$f(u_{i\,2j}) = (2mn-1)-n(i-1)-2(j-1), 1 \le j \le \frac{n-1}{2}$, $1 \le i \le m$.

The induced edge labeling are given as

$f^*(u_{m+1-i,n+1-j}\ u_{m+1-i,n-j}) = 2j+2n(i-1)$,

$1 \le j \le n-1, 1 \le i \le m$.

$f^*(u_{m+1-i,\frac{n+1}{2}}\ u_{m-i,\frac{n+1}{2}}) = 2ni$, $1 \le i \le m-1$

It shows the edge labels are distinct.

Hence, $P_{(n)_m}$ is an Odd-Even graceful graph.

For example the Odd-Even graceful labeling of $P_{(5)_5}$ is shown in *figure 8.*

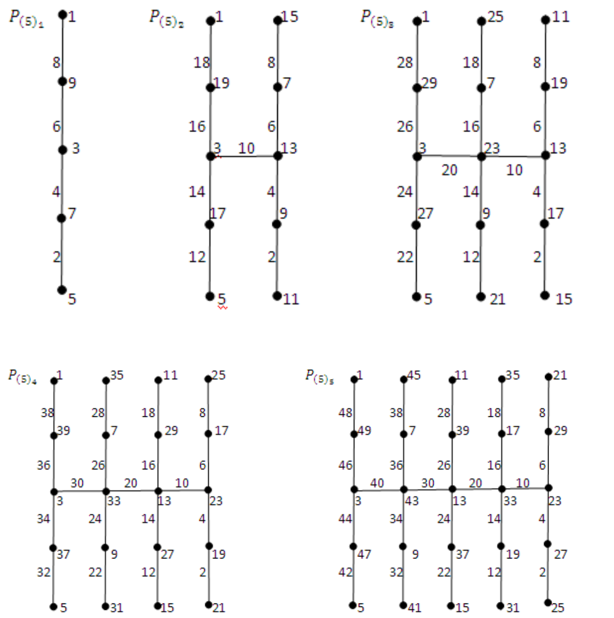

Fig.8. $P_{(5)_5}$

Theorem 2.19. $P_{(n)_m}$ $(n{:}\, even)$ is an Odd-Even graceful graph.

Proof: Let G = $P_{(n)_m}$

Let V(G) = $\{(u_{ij} : 1 \le i \le m, 1 \le j \le n)\}$.

Let E(G) = $\{(u_{ij}\ u_{ij+1} : 1 \le i \le m, 1 \le j \le n-1) \cup \left(\left(u_{i\,\frac{n+2}{2}}\ u_{i+1\,\frac{n}{2}}\right) : 1 \le i \le m-1\right)\}$ and

$|V(G)|$ = mn and $|E(G)|$ = mn − 1 .

Define f: V(G) → {1,3,5,…,(2nm−1)} as

$f(u_{i\,2j-1}) = n(i-1)+1+2(j-1), 1 \le j \le \frac{n}{2}$, $1 \le i \le m$

$f(u_{i\,2j}) = (2mn-1)-n(i-1)-2(j-1), 1 \le j \le \frac{n}{2}$, $1 \le i \le m$.

The induced edge labeling are given as

$f^*(u_{m+1-i,n+1-j}\ u_{m+1-i,n-j}) = 2j+2n(i-1)$, $1 \le j \le n-1, 1 \le i \le m$

$f^*(u_{m+1-i,\frac{n}{2}}\ u_{m-i,\frac{n+2}{2}}) = 2ni$, $1 \le i \le m-1$

It shows the edge labels are distinct.

Hence, $P_{(n)_m}$ is an Odd-Even graceful graph.

For example the Odd-Even graceful labeling of $P_{(6)_3}$ is shown in *figure 9.*

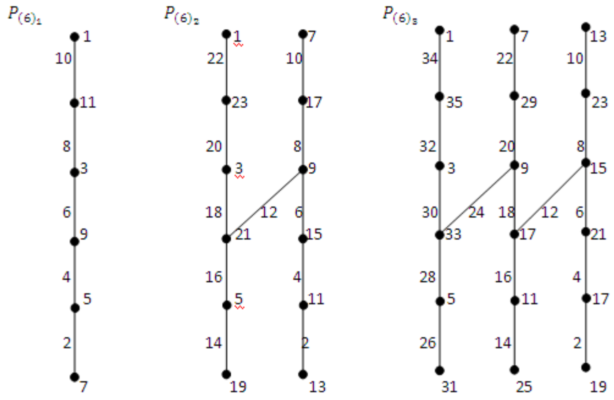

Fig.9. $P_{(6)_3}$

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