International Journal of Innovation in Science and Mathematics
Volume 2, Issue 5, ISSN (Online): 2347–9051

Fuzzy Translation and Fuzzy Multiplication on PS-Algebras

T. Priya
Department of Mathematics,
PSNA College of Engineering and Technology
Dindigul – 624 622, Tamilnadu, India
Email: tpriyasuriya@gmail.com

T. Ramachandran
Department of Mathematics,
M. V. Muthiah Government Arts College for Women,
Dindigul– 624 001, Tamilnadu, India
Email: yasrams@yahoo.co.in

Abstract **– In this paper, we define a fuzzy translation and fuzzy multiplication on PS-algebras and discussed some of their properties in detail by using the concepts of fuzzy PS-ideal and fuzzy PS-sub algebra.**

Keywords **– Fuzzy-α-Translation, Fuzzy-α-Multiplication of Fuzzy PS-Algebra, Fuzzy PS-Ideal, Fuzzy PS-Sub Algebra, Fuzzy Extension.**

AMS Subject Classification (2000): **06F35, 03G25.**

I. Introduction

The concept of fuzzy set was initiated by L.A.Zadeh in 1965 [4]. It has opened up keen insights and applications in a wide range of scientific fields. Since its inception, the theory of fuzzy subsets has developed in many directions and found applications in a wide variety of fields. The study of fuzzy subsets and its applications to various mathematical contexts has given rise to what is now commonly called fuzzy mathematics. Fuzzy algebra is an important branch of fuzzy mathematics. Fuzzy ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. In this way , K.Iseki and S.Tanaka [1] introduced the concept of BCK-algebras in 1978. K.Iseki [2] introduced the concept of BCI-algebras in 1980. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. T.Priya and T.Ramachandran [6][7] introduced the class of PS-algebras , which is a generalization of BCI / BCK/Q / KU / d algebras. In this paper, we introduce the concept of fuzzy-α-translation ,fuzzy-α-multiplication of fuzzy PS-algebras and fuzzy extensions and established some of its properties in detail.

II. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1 [1]

A BCK- algebra is an algebra $(X,*,0)$ of type(2,0) satisfying the following conditions:

i) $(x * y) * (x * z) \leq (z * y)$

ii) $x * (x * y) \leq y$

iii) $x \leq x$

iv) $x \leq y$ and $y \leq x \Rightarrow x=y$

v) $0 \leq x \Rightarrow x=0$, where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in X$.

Definition 2.2 [2]

A BCI- algebra is an algebra $(X,*,0)$ of type(2,0) satisfying the following conditions:

i) $(x * y) *(x * z) \leq (z*y)$

ii) $x * (x * y) \leq y$

iii) $x \leq x$

iv) $x \leq y$ and $y \leq x \Rightarrow x = y$

v) $x \leq 0 \Rightarrow x = 0$, where $x \leq y$ is defined by $x * y = 0$,for all $x, y, z \in X$.

Definition 2.3:[7]

A nonempty set X with a constant 0 and a binary operation '$*$' is called PS – algebra if it satisfies the following axioms.

1. $x * x = 0$
2. $x * 0 = 0$
3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$, $\forall x ,y \in X$.

Definition 2.4: [7]

Let S be a non empty subset of a PS -algebra X , then S is called a PS-sub algebra of X if $x * y \in S$,for all $x ,y \in S$.

Definition 2.5:[7]

Let X be a PS-algebra and I be a subset of X, then I is called a PS-ideal of X if it satisfies following conditions:

1. $0 \in I$
2. $y * x \in I$ and $y \in I \Rightarrow x \in I$

Definition 2.6:[6]

Let X be a PS-algebra. A fuzzy set μ in X is called a fuzzy PS-ideal of X if it satisfies the following conditions.

i) $\mu(0) \geq \mu(x)$

ii) $\mu (x) \geq \min \{\mu (y *x), \mu(y)\}$, for all $x, y \in X$

Definition 2.7:[6]

A fuzzy set μ in a PS-algebra X is called a fuzzy PS-sub algebra of X if

$\mu(x * y) \geq \min \{\mu(x), \mu(y)\}$, for all $x, y \in X$.

III. Fuzzy Translation and Fuzzy Multiplication in PS-Algebra

Let X be a PS-algebra. For any fuzzy set μ of X, we define $T = 1- \sup\{\mu(x) / x \in X \}$, unless otherwise we specified.

Definition 3.1:([3][5])

Let μ be a fuzzy subset of X and $\alpha \in [0,T]$. A mapping $\mu_\alpha^T: X \to [0, 1]$ is said to be a fuzzy-α-translation of μ if it satisfies $\mu_\alpha^T (x) = \mu(x) + \alpha$, $\forall x \in X$.

Definition 3.2:([3][5])

Let μ be a fuzzy subset of X and $\alpha \in [0,1]$. A mapping $\mu_\alpha^M : X \to [0, 1]$ is said to be a fuzzy-α-multiplication of μ if it satisfies $\mu_\alpha^M (x) = \alpha\, \mu(x)$, $\forall x \in X$.

Example 3.3: Let X = { 0, 1, 2 } be the set with the following table.

*	0	1	2
0	0	1	2
1	0	0	1
2	0	2	0

Then (X, $*$, 0) is a PS – algebra.

Define a fuzzy set μ of X by $\mu(x) = \begin{cases} 0.3 \ \text{if} \ x \neq 1 \\ 0.2 \ \text{if} \ x = 1 \end{cases}$. Then μ is a fuzzy PS-sub algebra of X. Here T = 1 – sup {μ(x) / x ∈ X} = 1 – 0.3 = 0.7, Choose α= 0.4 ∈[0, T] and β = 0.5∈[0, 1].

Then the mapping $\mu_{0.4}^{T}$: X →[0, 1] is defined by

$$\mu_{0.4}^{T} = \begin{cases} 0.3+0.4=0.7 \ \text{if} \ x \neq 1 \\ 0.2+0.4=0.6 \ \text{if} \ x = 1 \end{cases}$$

which satisfies $\mu_{0.4}^{T}(x) = \mu(x) + 0.4$, ∀ x ∈ X, is a fuzzy 0.4-translation and the mapping

$\mu_{0.5}^{M}$: X →[0, 1] is defined by $\mu_{0.5}^{M}$ (x) =

$$\begin{cases} (0.5)(0.3) = 0.15 \ \text{if} \ x \neq 1 \\ (0.5)(0.2) = 0.10 \ \text{if} \ x = 1 \end{cases}$$

which satisfies $\mu_{0.5}^{M}(x) = (0.5)\mu(x)$, ∀ x ∈ X, is a fuzzy 0.5-multiplication.

Theorem 3.4:

If μ of X is a fuzzy PS- sub algebra and α ∈ [0,T] , then the fuzzy-α-translation μ_α^{T} (x) of μ is also a fuzzy PS- sub algebra of X.

Proof :

Let x, y ∈ X and α ∈ [0,T].

Then μ(x * y) ≥ min{μ(x), μ(y)}

Now μ_α^{T} (x * y) = μ(x * y) + α
≥ min{μ(x), μ(y)} + α
= min{μ(x) + α, μ(y) + α}
= min{ μ_α^{T} (x) , μ_α^{T} (y)}

Theorem 3.5:

Let μ be a fuzzy subset of X such that the fuzzy-α-translation μ_α^{T} (x) of μ is a fuzzy sub algebra of X for some α ∈ [0,T], then μ is a fuzzy sub algebra of X.

Proof:

Assume that μ_α^{T} is a fuzzy subalgebra of X for some α ∈ [0,T].

Let x , y ∈ X. We have

μ(x * y) + α = μ_α^{T} (x * y)
≥ min { μ_α^{T} (x) , μ_α^{T} (y)}
= min {μ(x) + α, μ(y) + α}
= min{μ(x), μ(y)} + α

⇒ μ(x * y) ≥ min{μ(x), μ(y)} for all x, y ∈ X. Hence μ is fuzzy sub algebra of X.

Theorem 3.6:

For any fuzzy PS- sub algebra μ of X and α ∈ [0,T], if the fuzzy-α-multiplication μ_α^{M} (x) of μ is a fuzzy PS- sub algebra of X.

Proof:

Let x, y ∈ X and α ∈ [0,T].

Then μ(x * y) ≥ min {μ(x), μ(y)}

Now,

μ_α^{M} (x * y) = α μ(x * y)
≥ α min {μ(x), μ(y)}
= min {α μ(x), α μ(y)}
= min { μ_α^{M} (x) , μ_α^{M} (y)}

⇒ μ_α^{M} (x * y) ≥ min{ μ_α^{M} (x) , μ_α^{M} (y)}

∴ μ_α^{M} is a fuzzy PS-sub algebra of X.

Theorem 3.7:

For any fuzzy subset μ of X and α ∈ [0,T],if the fuzzy-α-multiplication μ_α^{M} (x) of μ is a fuzzy PS- sub algebra of X, then so is μ.

Proof :

Assume that μ_α^{M} (x) of μ is a fuzzy PS- sub algebra of X for some α ∈ [0,T].

Let x , y ∈ X. We have

α μ(x * y) = μ_α^{M} (x * y)
≥ min{ μ_α^{M} (x) , μ_α^{M} (y)}
= min {α μ(x), α μ(y)}
= α min {μ(x), μ(y)}

⇒ μ(x * y) ≥ min {μ(x), μ(y)}. Hence μ is a fuzzy PS-sub algebra of X.

Theorem 3.8:

If the fuzzy-α-translation μ_α^{T} (x) of μ is a fuzzy PS-ideal, then it satisfies the condition μ_α^{T}(x * (y * x)) ≥ μ_α^{T} (y).

Proof :

μ_α^{T} (x * (y * x)) = μ(x * (y * x)) + α
≥ min{ μ(y *(x * (y * x))) + α ,μ (y) + α}
= min { μ(0) + α, μ (y) + α }
≥ min { μ_α^{T} (0), μ_α^{T} (y) }
= μ_α^{T} (y).

Theorem 3.9:

If μ is a fuzzy PS-ideal of X, then the fuzzy α-translation μ_α^{T} of μ is a fuzzy PS-ideal of X, for all α ∈ [0,T].

Proof :

Let μ be a fuzzy PS-ideal of X and let α ∈ [0,T].

Then

μ_α^{T}(0) = μ(0) + α
≥ μ(x) + α
= μ_α^{T}(x)

And μ_α^{T}(x) = μ(x) + α
≥ min { μ(y*x), μ(y)} + α
= min { μ((y*x) + α, μ(y) + α}
= min { μ_α^{T}(y*x) , μ_α^{T}(y) }

Hence μ_α^{T} of μ is a fuzzy PS-ideal of X , ∀ α ∈[0, T]

Theorem 3.10:

Let μ be a fuzzy subset of X such that the fuzzy α-translation μ_α^{T} of μ is a fuzzy PS-ideal of X for some α ∈ [0,T], then μ is a fuzzy PS-ideal of X.

Proof :

Assume that μ_α^{T} is a fuzzy PS-ideal of X for some α ∈ [0,T]. Let x, y ∈ X.

Then μ(0) + α = μ_α^{T}(0)
≥ μ_α^{T}(x)
= μ(x) + α

and so μ(0) ≥ μ(x)

International Journal of Innovation in Science and Mathematics
Volume 2, Issue 5, ISSN (Online): 2347–9051

Also, $\mu(x) + \alpha = \mu_\alpha^T(x)$

$\geq \min \{ \mu_\alpha^T(y*x) , \mu_\alpha^T(y) \}$

$= \min\{ \mu(y*x) + \alpha, \mu(y) + \alpha\}$

$= \min\{ \mu((y*x), \mu(y)\} + \alpha$

and so $\mu(x) \geq \min\{ \mu(y*x), \mu(y)\}$

Hence μ is a fuzzy PS-ideal of X.

Theorem 3.11:

Let $\alpha \in [0,T]$ and let μ be a fuzzy PS-ideal of X. If X is a PS-algebra, then the fuzzy α - translation μ_α^T of μ is a fuzzy PS- sub algebra of X.

Proof :

Let x, y $\in$ X .

Now, we have

$\mu_\alpha^T(x*y) = \mu(x*y) + \alpha$

$\geq \min \{\mu(y * (x * y)), \mu(y)\} + \alpha$

$\geq \min \{\mu(0) , \mu(y)\} + \alpha$

$\geq \min \{\mu(x) , \mu(y)\} + \alpha$

$= \min \{\mu(x) + \alpha , \mu(y) + \alpha\}$

$= \min \{\mu_\alpha^T(x) , \mu_\alpha^T(y) \}$

Hence μ_α^T is a fuzzy PS- sub algebra of X.

Theorem 3.12:

If the fuzzy α - translation μ_α^T of μ is a fuzzy PS- ideal of X, $\alpha \in [0,T]$, then μ is a fuzzy PS- sub algebra of X.

Proof:

Let us assume that μ_α^T of μ is a fuzzy PS- ideal of X. Then

$\mu(x * y) + \alpha = \mu_\alpha^T (x * y)$

$\geq \min \{\mu_\alpha^T (y * (x * y)), \mu_\alpha^T (y)\}$

$\geq \min \{\mu_\alpha^T (0) , \mu_\alpha^T (y)\}$

$\geq \min \{\mu_\alpha^T(x) , \mu_\alpha^T(y) \}$

$= \min \{\mu(x) + \alpha , \mu(y) + \alpha\}$

$= \min \{\mu(x) , \mu(y)\} + \alpha$

$\Rightarrow \mu(x * y) \geq \min \{\mu(x) , \mu(y)\}$

Hence μ is a fuzzy PS- sub algebra of X.

Theorem 3.13:

Let μ be a fuzzy subset of X such that the fuzzy α-multiplication μ_α^M of μ is a fuzzy PS-ideal of X for some $\alpha \in (0,1]$, then μ is a fuzzy PS-ideal of X.

Proof :

Assume that μ_α^M is a fuzzy PS-ideal of X for some $\alpha \in (0,1]$. Let x, y $\in$ X.

Then $\alpha\, \mu(0) = \mu_\alpha^M (0)$

$\geq \mu_\alpha^M (x)$

$= \alpha\, \mu(x)$

and so $\mu(0) \geq \mu(x)$

Also, $\alpha\, \mu(x) = \mu_\alpha^M(x)$

$\geq \min \{ \mu_\alpha^M (y * x) , \mu_\alpha^M (y) \}$

$= \min \{ \alpha\, \mu(y * x) , \alpha\, \mu(y) \}$

$= \alpha \min \{ \mu(y * x), \mu(y)\}$

and so $\mu(x) \geq \min\{ \mu(y*x), \mu(y)\}$

Hence μ is a fuzzy PS-ideal of X.

Theorem 3.14:

If μ is a fuzzy PS-ideal of X, then the fuzzy α-multiplication μ_α^M of μ is a fuzzy PS-ideal of X, for all $\alpha \in (0,1]$.

Proof :

Let μ be a fuzzy PS-ideal of X and let $\alpha \in (0,1]$. Then

$\mu_\alpha^M(0) = \alpha\, \mu(0)$

$\geq \alpha\, \mu(x)$

$= \mu_\alpha^M(x)$

And $\mu_\alpha^M(x) = \alpha\, \mu(x)$

$\geq \alpha \min \{ \mu(y * x), \mu(y)\}$

$= \min \{ \alpha\, \mu(y * x), \alpha\, \mu(y) \}$

$= \min \{ \mu_\alpha^M(y * x) , \mu_\alpha^M(y) \}$

Hence μ_α^M of μ is a fuzzy PS-ideal of X , $\forall\, \alpha \in (0, 1]$.

Theorem 3.15:

Let $\alpha \in [0,1]$ and let μ be a fuzzy PS-ideal of a PS-algebra X. Then the fuzzy α - multiplication μ_α^M of μ is a fuzzy PS- sub algebra of X.

Proof :

Let x, y $\in$ X .

Now, we have

$\mu_\alpha^M (x * y) = \alpha\, \mu(x*y)$

$\geq \alpha \min \{\mu(y * (x * y)), \mu(y)\}$

$\geq \alpha \min \{\mu(0), \mu(y)\}$

$\geq \alpha \min \{\mu(x), \mu(y)\}$

$= \min \{\alpha\, \mu(x), \alpha\, \mu(y)\}$

$= \min \{\mu_\alpha^M(x), \mu_\alpha^M(y)\}$

Hence μ_α^M is a fuzzy PS- sub algebra of X.

Theorem 3.16:

If the fuzzy α - multiplication μ_α^M of μ is a fuzzy PS-ideal of X, $\alpha \in [0,1]$, then μ is a fuzzy PS- sub algebra of X.

Proof:

Let us assume that μ_α^M of μ is a fuzzy PS- ideal of X. Then

$\alpha\, \mu(x * y) = \mu_\alpha^M (x * y)$

$\geq \min \{\mu_\alpha^M (y * (x * y)), \mu_\alpha^M (y)\}$

$\geq \min \{\mu_\alpha^M (0), \mu_\alpha^M (y)\}$

$\geq \min \{\mu_\alpha^M (x), \mu_\alpha^M (y) \}$

$= \min \{\alpha\, \mu(x),\alpha\, \mu(y)\}$

$= \alpha \min \{\mu(x) , \mu(y)\}$

$\Rightarrow \mu(x * y) \geq \min \{\mu(x) , \mu(y)\}$

Hence μ is a fuzzy PS- sub algebra of X.

Theorem 3.17:

Intersection and union of any two fuzzy translations of a fuzzy PS-ideal μ of X is also a fuzzy PS – ideal of X.

Proof :

Let μ_α^T and μ_ν^T be two fuzzy translations of a fuzzy PS-ideal μ of X, where $\alpha,\nu \in [0, T]$.

Assume that $\alpha \leq \nu$. Then by theorem 3.14, μ_α^T and μ_ν^T are fuzzy PS-ideals of X.

Now, $(\mu_\alpha^T \cap \mu_\nu^T) (x) = \min \{ \mu_\alpha^T (x), \mu_\nu^T (x) \}$

$= \min \{ \mu(x) + \alpha, \mu(x) + \nu \}$

$= \mu(x) + \alpha$

$= \mu_\alpha^T (x)$

And $(\mu_\alpha^T \cup \mu_\nu^T) (x) = \max \{ \mu_\alpha^T (x), \mu_\nu^T (x) \}$

$= \max \{ \mu(x) + \alpha, \mu(x) + \nu \}$

$= \mu(x) + \nu$

$= \mu_\nu^T (x)$

Hence $\mu_\alpha^T \cap \mu_\nu^T$ and $\mu_\alpha^T \cup \mu_\nu^T$ are fuzzy PS- ideals of X.

International Journal of Innovation in Science and Mathematics
Volume 2, Issue 5, ISSN (Online): 2347–9051

IV. Fuzzy Extensions of PS-Ideals of PS-Algebras

In this section, we introduced the of fuzzy extensions of PS-ideals of PS-algebras and proved some standard results.

Definition 4.1:

Let μ_1 and μ_2 be two fuzzy sets of X such that μ_2 is a fuzzy extension of μ_1. If μ_1 is a fuzzy PS-ideal of X implies that μ_2 is a fuzzy PS-ideal of X, then μ_2 is called as fuzzy PS-ideal extension of μ_1.

Example 4.2: Consider the PS-algebra as in example 3.3, the fuzzy sets μ_1 and μ_2 of X is defined as follows :

$$\mu_1(x) = \begin{cases} 0.7 & \text{if } x = 0 \\ 0.5 & \text{if } x \neq 0 \end{cases} \quad \text{and } \mu_2(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.7 & \text{if } x = 1 \\ 0.5 & \text{if } x = 2 \end{cases}$$

are fuzzy PS-ideals of X. It is clear that μ_2 is a fuzzy PS-ideal extension of μ_1.

Theorem 4.3:

Intersection of any two fuzzy PS-ideal extensions of a fuzzy PS-ideal μ of X is a fuzzy PS-ideal extension of μ.

Proof :

Let μ_1 and μ_2 be two fuzzy PS-ideal extensions of a fuzzy PS-ideal μ of X. Then $\mu_1(x) \geq \mu(x)$ and $\mu_2(x) \geq \mu(x)$, for all $x \in X$. Since μ is a fuzzy PS-ideal of X, μ_1 and μ_2 are fuzzy PS-ideals of X. Then $\mu_1 \cap \mu_2$ is also a fuzzy PS-ideal of X (By theorem 3.4[6]).Now $(\mu_1 \cap \mu_2)(x) = \min \{\mu_1(x), \mu_2(x)\} \geq \min \{\mu(x), \mu(x)\} = \mu(x)$. Hence $\mu_1 \cap \mu_2$ is a fuzzy PS-ideal extension of μ.

Remark :

Union of any two fuzzy PS-ideal extensions of a fuzzy PS-ideal μ of X need not be a fuzzy PS-ideal extension of μ.

Consider the example 3.3, the fuzzy sets μ, μ_1 and μ_2 of X is defined as follows :

$$\mu(x) = \begin{cases} 0.6 & \text{if } x = 0 \\ 0.5 & \text{if } x \neq 0 \end{cases} ; \quad \mu_1(x) = \begin{cases} 0.7 & \text{if } x = 0 \\ 0.5 & \text{if } x \neq 0 \end{cases} \quad \text{and}$$

$$\mu_2(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.7 & \text{if } x = 1 \\ 0.5 & \text{if } x = 2 \end{cases}$$

From the above definition itself it is clear that μ_1 and μ_2 are fuzzy PS-ideal extensions of the fuzzy PS-ideal μ. Here $(\mu_1 \cup \mu_2)(0) = \max \{\mu_1(0), \mu_2(0)\} = 0.8$; $(\mu_1 \cup \mu_2)(1) = 0.7$; $(\mu_1 \cup \mu_2)(2) = 0.5$, is not a fuzzy PS-ideal extension, since $(\mu_1 \cup \mu_2)(2) = 0.5 \geq 0.7 = \min \{(\mu_1 \cup \mu_2)(1*2), (\mu_1 \cup \mu_2)(1)\} = \min\{(\mu_1 \cup \mu_2)(1), (\mu_1 \cup \mu_2)(1)\}$.

Theorem 4.4 :

Let μ be a fuzzy PS-ideal of X. The fuzzy α - translation μ_α^T is a fuzzy PS-ideal extension of μ , for all $\alpha \in [0,T]$.

Proof :

If μ is a fuzzy PS-ideal of X, then by theorem 3.11,the fuzzy α- translation μ_α^T of μ is also a fuzzy PS-ideal of X , for all $\alpha \in [0,T]$. Now $\mu_\alpha^T(x) = \mu(x) + \alpha \geq \mu(x)$, for all $x \in X$. Hence, the fuzzy α - translation μ_α^T is a fuzzy PS-ideal extension of μ.

Theorem 4.5 :

Let μ be a fuzzy PS-ideal of X. If $\alpha \geq \delta$, with $\alpha, \delta \in [0,T]$, then the fuzzy α-translation μ_α^T of μ is a fuzzy PS-ideal extension of the fuzzy δ-translation μ_δ^T of μ.

Proof:

Let μ be a fuzzy PS-ideal of X. Then by theorem 3.11, the fuzzy α- translation μ_α^T of μ and the fuzzy δ-translation μ_δ^T of μ are fuzzy PS-ideals of X, for all α, $\delta \in [0,T]$. Since $\alpha \geq \delta$, $\mu(x) + \alpha \geq \mu(x) + \delta$, for all $x \in X$. Therefore $\mu_\alpha^T(x) \geq \mu_\delta^T(x)$. Hence μ_α^T is a fuzzy PS-ideal extension of μ_δ^T.

Theorem 4.6 :

Let μ be a fuzzy set of X, $\alpha \in [0,T]$ and $\delta \in (0,1]$. If the fuzzy-δ-multiplication $\mu_\delta^M(x)$ of μ is a fuzzy PS- ideal of X, then the fuzzy-α-translation $\mu_\alpha^T(x)$ of μ is a fuzzy PS-ideal extension of μ_δ^M.

Proof:

Let $\alpha \in [0,T]$, $\delta \in (0,1]$ and $\mu_\delta^M(x)$ of μ is a fuzzy PS-ideal of X. Then by theorem 3.13, μ is a fuzzy PS- ideal of X. By theorem 3.9, $\mu_\alpha^T(x)$ of μ is a fuzzy PS- ideal of X. Now, $\mu_\alpha^T(x) = \mu(x) + \alpha \geq \mu(x) \geq \mu(x)\,\delta = \mu_\delta^M(x)$.

Therefore, $\mu_\alpha^T(x)$ of μ is a fuzzy PS-ideal extension of μ_δ^M.

V. Conclusion

In this article authors have been discussed fuzzy translation and fuzzy multiplication on PS-algebras through PS-sub algebras and PS-ideals.It has been observed that PS-algebras as an another generalization of BCK/BCI/Q/d/TM/KU-algebras. Interestingly, fuzzy extensions of PS-ideals of PS-algebras has been studied, which adds an another dimension to the defined PS-algebras. This concept can further be generalized to Intuitionistic fuzzy set, interval valued fuzzy sets for new results in our future work.

Acknowledgement

Authors wish to thank Dr. K.T. Nagalakshmi , Professor and Head ,Department of Mathematics, K L N College of Information and Technology, Pottapalayam, Sivagangai District, Tamilnadu, India, Prof. P.M.Sithar Selvam, Professor and Head , Department of Mathematics, RVS School of Engineering and Technology, Dindigul, Tamilnadu, India, for their help to make this paper as successful one.

References

[1] K.Iseki and S.Tanaka , An introduction to the theory of BCK – algebras , Math Japonica 23 (1978), 1- 20 .

[2] K.Iseki , On BCI-algebras , Math.Seminar Notes 8 (1980), 125-130.

[3] Kyoung Ja Lee, Young Bae Jun, and Myung Im Doh, Fuzzy Translations And Fuzzy Multiplications Of BCK/BCI-Algebras,Commun. Korean Math. Soc. 24 (2009), No. 3, 353-360.

[4] L.A.Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.

[5] M.Abu Ayub Ansari and M.Chandramouleeswaran, Fuzzy translations of fuzzy β-ideals of β -algebras, International Journal of Pure and Applied Mathematics,Vol.92 No.5,2014, 657- 667.

[6] T.Priya and T.Ramachandran , A note on Fuzzy PS –Ideals in PS-Algebra and its level subsets, International Journal of Advanced Mathematical Sciences, Vol.2, No.2, 2014, 101-106.

[7] T.Priya and T.Ramachandran , Classification of PS-algebras, International Journal of Innovative Science,Engineering and Technology,Vol.1, No.7,2014,193-199.

AUTHOR'S PROFILE

T. Priya

is working as an Associate Professor of Mathematics, at P. S.N. A College of Engineering & Technology, Dindigul, Tamilnadu, India with 12 years of experience, published a book with Scitech Publishers, and authored more than 15 papers in various national and international journals.

Dr. T. Ramachandran

is working as an Assistant Professor of Mathematics at M.V.M Government Arts and Science College for Women, Dindigul, Tamilnadu, India with 30 years of experience, published two books with Tata Mcgraw Hill Publishers, and authored more than 30 papers in various national and international journals. He has guided 28 M.Phil scholars, and one Ph.D scholar. 4 M.Phil and 5 Ph.D Scholars are doing their research under his guidance.