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Plane Symmetric Cosmological Model with Interacting Dark Matter and Holographic Dark Energy using Special form of Deceleration Parameter

V. B. Raut, K. S. Adhav*, D. K. Joshi
Department of Mathematics,
Sant Gadge Baba Amravati University, Amravati (444602), Maharashtra, India
*Email: ati_ksadhav@yahoo.co.in

Abstract **– In this paper, we have solved the Einstein's field equations for the space time described by a plane symmetric metric with interacting Dark Matter and Holographic Dark Energy. The solutions of the field equations have been obtained under the assumption of a special form of deceleration parameter. The physical and geometrical aspects of the model are also discussed.**

Keywords **– Plane Symmetric Cosmological Space-Time, Interacting Dark Fluids, A Special Form of Deceleration Parameter, State Finder Parameters, Coincidence Problem.**

I. Introduction

We know that there are many candidates for dark energy namely the quintessence scalar field models, K-essence models, tachyon field models, quintom field models. A special class of cosmological models in which holographic Dark Energy is allowed to interact with Dark Matter are studied by many authors [1-9]. Non-interacting holographic dark energy models with linearly varying deceleration parameter for Bianchi type-I and V universe and interacting holographic dark energy model for Bianchi type-II are studied by Sarkar [10-12].

With this motivation, in this paper, we have considered the plane symmetric cosmological model filled with interacting dark matter and holographic dark energy. The solutions of the field equations have been obtained under the assumption of a special form of deceleration parameter. The physical and geometrical aspects of the model are also discussed.

II. Metric and Field Equations

In view of the importance of the plane symmetry which has been exploited to study the cosmological models by [13-18], we have considered the line element in plane symmetric form as

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 , \qquad (2.1)$$

where A and B are the scale factors and functions of the cosmic time t only.

The Einstein's field equations are ($8\pi G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2} g_{ij} R = -{}^{m}T_{ij} + {}^{\Lambda}T_{ij} \quad , \qquad (2.2)$$

where ${}^{m}T_{ij} = \rho_m u_i u_j$ and

$${}^{\Lambda}T_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j + g_{ij} p_\Lambda \qquad (2.3)$$

are matter tensors for dark matter (pressureless i.e. $w_m = 0$) and holographic dark energy. Here ρ_m is the energy density of dark matter and ρ_Λ and p_Λ are the energy density and pressure of holographic dark energy.

The Einstein's field equations (2.2) for metric (2.1) with the help of equations (2.3) can be written as

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = \rho_m + \rho_\Lambda , \qquad (2.4)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -p_\Lambda , \qquad (2.5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p_\Lambda , \qquad (2.6)$$

where overhead dot (·) represents derivative with respect to time *t*.

Assuming that both components of energy do not conserve separately but interact with each other in such a manner that the balance equations take the form

$$\dot{\rho}_m + \left(\frac{\dot{V}}{V}\right)\rho_m = Q , \qquad (2.7)$$

$$\dot{\rho}_\Lambda + \left(\frac{\dot{V}}{V}\right)(1 + w_\Lambda)\rho_\Lambda = -Q \ , \qquad (2.8)$$

where $w_\Lambda = p_\Lambda / \rho_\Lambda$ is the equation of state parameter for holographic dark energy and $Q > 0$ measures the strength of the interaction.

The interaction between dark energy and dark matter could be expressed phenomenologically in the form (Guo *et al.* and Amendola *et al*. [19,20])

$$Q = 3b^2 H\rho_m = b^2 \frac{\dot{V}}{V}\rho_m , \qquad (2.9)$$

where b^2 is coupling constant.

Cai & Wang [21] have taken same relation for interacting phantom dark energy and dark matter in order to avoid the coincidence problem.

From equations (2.13) and (2.11), we get the energy density of dark matter as

$$\rho_m = \rho_0 V^{(b^2-1)} , \qquad (2.10)$$

where $\rho_0 > 0$ is a real constant of integration

Using equations (2.9) and (2.10), we get the interacting term as

$$Q = 3\rho_0 b^2 H V^{(b^2-1)} \ . \qquad (2.11)$$

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III. Solutions of the Field Equations

The field equations (2.4)-(2.6) is a system of three non-linear differential equations with four unknowns A, B, ρ_m and ρ_Λ. In order to solve the system completely, we use a special form of deceleration parameter defined by Singh and Debnath [22] for FRW metric as

$$q=-\frac{\ddot{a}a}{\dot{a}^2}=-1+\frac{\alpha}{1+a^{\alpha}}, \tag{3.1}$$

where $\alpha>0$ is a constant and a is scale factor of the universe.

After solving equation (3.1) one can obtain the mean Hubble parameter H as

$$H=\frac{\dot{a}}{a}=k(1+a^{-\alpha}), \tag{3.2}$$

where $k>0$ is a constant of integration.

On integrating equation (3.2), we obtain the mean scale factor as

$$a=(e^{k\alpha t}-1)^{1/\alpha}. \tag{3.3}$$

We define the spatial volume V and average scale factor a as

$$V=A^2B \quad \text{and} \quad a=(A^2B)^{1/3}. \tag{3.4}$$

The mean Hubble parameter H is defined as

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(H_x+H_y+H_z\right), \tag{3.5}$$

where $H_x=\frac{\dot{A}}{A}$, $H_y=\frac{\dot{A}}{A}$, $H_z=\frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of x, y and z axes respectively.

Subtracting equation (2.6) from equation (2.5) and using equation (3.4), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{V}}{V}=0. \tag{3.6}$$

After solving equation (3.6), we can write the metric functions A and C explicitly as

$$A=c_2^{1/3}\, a\, \exp\left(\frac{c_1}{3}\int\frac{dt}{V}\right), \tag{3.7}$$

$$B=c_2^{-2/3}\, a\, \exp\left(-\frac{2c_1}{3}\int\frac{dt}{V}\right), \tag{3.8}$$

where c_1 and c_2 are constants of integration.

Using equation (3.3) for $k=1$ in equation (3.7) and equation (3.8), we obtain the exact value of scale factors as

$$A=c_2^{1/3}(e^{\alpha t}-1)^{\frac{1}{\alpha}}\exp\left\{-\frac{c_1}{9}\left[\begin{array}{r}(1-e^{-\alpha t})^{3/\alpha}(e^{\alpha t}-1)^{-3/\alpha}\\ {}_2F_1\left(\frac{3}{\alpha},\frac{3}{\alpha};\frac{\alpha+3}{\alpha};e^{-\alpha t}\right)\end{array}\right]\right\} \tag{3.9}$$

$$B=c_2^{-2/3}(e^{\alpha t}-1)^{\frac{1}{\alpha}}\exp\left\{\frac{2c_1}{9}\left[\begin{array}{r}(1-e^{-\alpha t})^{3/\alpha}(e^{\alpha t}-1)^{-3/\alpha}\\ {}_2F_1\left(\frac{3}{\alpha},\frac{3}{\alpha};\frac{\alpha+3}{\alpha};e^{-\alpha t}\right)\end{array}\right]\right\} \tag{3.10}$$

where ${}_2F_1(l,m;n;t)$ is hypergeometric function.

The directional Hubble parameters in the directions of x, y and z -axis are found to be

$$H_x=H_y=\frac{e^{\alpha t}}{(e^{\alpha t}-1)}+\frac{c_1}{3}(e^{\alpha t}-1)^{-3/\alpha}, \tag{3.11}$$

$$H_z=\frac{e^{\alpha t}}{(e^{\alpha t}-1)}-\frac{2c_1}{3}(e^{\alpha t}-1)^{-3/\alpha}. \tag{3.12}$$

The mean Hubble parameter H is found to be

$$H=\frac{e^{\alpha t}}{(e^{\alpha t}-1)}. \tag{3.13}$$

The volume V is given by

$$V=(e^{\alpha t}-1)^{3/\alpha}. \tag{3.14}$$

The expansion scalar $\theta=3H$ is given by

$$\theta=\frac{3e^{\alpha t}}{(e^{\alpha t}-1)}. \tag{3.15}$$

The mean anisotropy parameter Δ of the expansion is define as

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2,$$

where $H_i(i=1,2,3)$ represent the directional Hubble parameters and the anisotropy parameter Δ of the expansion is found to be

$$\Delta=\frac{2c_1^2}{9}\frac{(e^{\alpha t}-1)^{2(\alpha-3)/\alpha}}{e^{2\alpha t}}. \tag{3.16}$$

The shear scalar is define as

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-3H^2\right) \text{ and found to be}$$

$$\sigma^2=\frac{c_1^2}{3}(e^{\alpha t}-1)^{-6/\alpha}. \tag{3.17}$$

The deceleration parameter is define as

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \text{ and found to be}$$

$$q=2e^{-\alpha t}-1. \tag{3.18}$$

Using equation (3.14) in equations (2.10) and (2.11), we get

$$\rho_m=\rho_0(e^{\alpha t}-1)^{\frac{3(b^2-1)}{\alpha}}, \tag{3.19}$$

$$Q=3\rho_0 b^2 e^{\alpha t}(e^{\alpha t}-1)^{\frac{3(b^2-1)}{\alpha}-1}. \tag{3.20}$$

Using equations (3.9)-(3.11) in the equation (2.4), we obtain the energy density of holographic dark energy as

$$\rho_\Lambda=\frac{3e^{2\alpha t}}{(e^{\alpha t}-1)^2}+\frac{c_1}{3}(e^{\alpha t}-1)^{-6/\alpha}\left[\frac{c_1}{3}+\frac{e^{\alpha t}}{(e^{\alpha t}-1)^{(\alpha-3)/\alpha}}\right]-\rho_0(e^{\alpha t}-1)^{\frac{3(b^2-1)}{\alpha}} \tag{3.21}$$

Using equations (3.19),(3.21) and (3.14) in the linear combination of equations (2.4-2.6), we obtain the pressure of holographic dark energy as

$$p_\Lambda = \frac{1}{(e^{\alpha t}-1)^2}\left[3e^{2\alpha t} - 2(3-\alpha)\right] + \frac{c_1}{3}(e^{\alpha t}-1)^{-6/\alpha}\left[\frac{c_1}{3} + \frac{e^{\alpha t}}{(e^{\alpha t}-1)^{(\alpha-3)/\alpha}}\right] \quad (3.22)$$

The EoS parameter of holographic dark energy is given by

$$w_\Lambda = \frac{2\alpha + 3(e^{2\alpha t}-2) + \frac{c_1}{3}(e^{\alpha t}-1)^{2(\alpha-3)/\alpha}\left[\frac{c_1}{3} + \frac{e^{\alpha t}}{(e^{\alpha t}-1)^{(\alpha-3)/\alpha}}\right]}{3e^{2\alpha t} + \frac{c_1}{3}(e^{\alpha t}-1)^{2(\alpha-3)/\alpha}\left[\frac{c_1}{3} + \frac{e^{\alpha t}}{(e^{\alpha t}-1)^{(\alpha-3)/\alpha}}\right] - \rho_0 (e^{\alpha t}-1)^{\frac{3(b^2-1)+2\alpha}{\alpha}}} \quad (3.23)$$

The coincidence parameter $\bar{r}$ i.e. the ratio of dark matter energy density to the dark energy density is given by

$$\bar{r} = \frac{3\rho_0 (e^{\alpha t}-1)^{\frac{3(b^2-1)+2\alpha}{\alpha}}}{9e^{2\alpha t} + c_1 (e^{\alpha t}-1)^{2(\alpha-3)/\alpha}\left[\frac{c_1}{3} + \frac{e^{\alpha t}}{(e^{\alpha t}-1)^{(\alpha-3)/\alpha}}\right] - 3\rho_0 (e^{\alpha t}-1)^{\frac{3(b^2-1)+2\alpha}{\alpha}}} \quad (3.24)$$

IV. Statefinder Diagnostic

The statefinder pair $\{r, s\}$ has been defined by Sahni *et al*. [23] as

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)}$$

The statefinder is a 'geometrical' diagnostic in the sense that it depends upon the expansion factor and hence upon the metric describing space-time. The state finder parameter pair for the spatially flat Λ CDM model corresponds to a fixed point in the diagram $\{s, r\}\big|_{\Lambda CDM} = \{0, 1\}$. Departure of a given dark energy model from this fixed point provides a good way of establishing the 'distance' of this model from Λ CDM [24].

The statefinder parameters r and s for special form of deceleration parameter model are given by

$$r = e^{-2\alpha t}\left[\alpha^2 + (\alpha-3)\alpha e^{\alpha t} + e^{2\alpha t}\right] \quad \text{and}$$

$$s = \frac{e^{-\alpha t}\left[\alpha^2 + (\alpha-3)\alpha e^{\alpha t} + e^{2\alpha t}\right] - 1}{3\left[(\alpha - e^{\alpha t})\right]}.$$

V. Discussion & Conclusion

i) The deceleration parameter (q): The Figure-1 shows that the universe accelerates after an epoch of deceleration in the special form of deceleration parameter cosmological model.

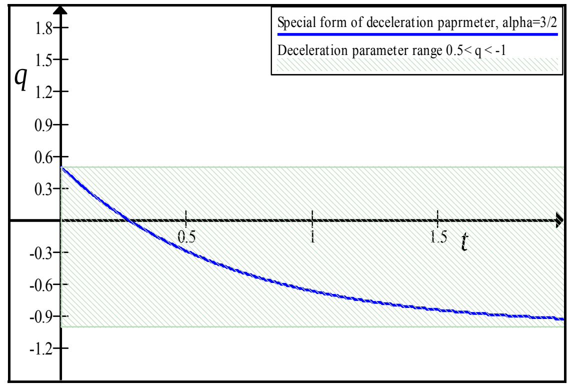

Fig.1. Evolution of deceleration parameter q.

For $\alpha = 3/2$ the deceleration parameter q is in the range $-1 \le q \le 0.5$ (shaded region in the Fig.1) which is consistent with the observations made by Perlmutter *et al.* and Riess *et al.* [25-27] indicating that the present day universe is undergoing accelerated expansion.

ii) The anisotropy parameter of expansion (Δ): In Figure-2 we plot anisotropy parameter of expansion Δ against cosmic time t for a special form of deceleration parameter cosmological model. It is observed that in the model anisotropy decreases to zero very quickly.

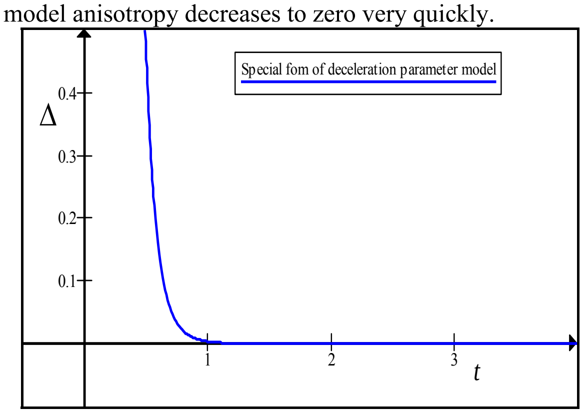

Fig.2. Evolution of anisotropy parameter of expansion Δ for $\alpha = 1$.

Hence, in the plane symmetric model obtained by using special form of deceleration parameter, the anisotropy of the model reaches to isotropy after some finite time which matches with the recent observations as the universe is isotropic at large scale.

iii) The equation of state parameter (w_Λ):

The Figure-3 shows the variation of EoS parameter (w_Λ) with cosmic time t for a special form of deceleration parameter cosmological model.

As per Wang *et al.* [3, 28] it has been argued that the interacting holographic dark energy model can accommodate the transition of the dark energy equation of state w from $w > -1$ to $w < -1$.

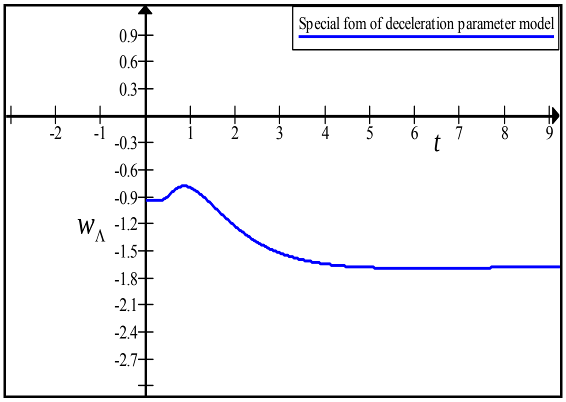

Fig.3. Evolution of EoS parameter (w_Λ).

Here, one should note that, Equation of State parameter (w_Λ) is reducing with respect to time t .

iv) Statefinder parameters (r, s):

The figure-4 represents the evolving trajectory of this scenario in the *s* - *r* plane.

This is quite different from those of other Dark Energy models.

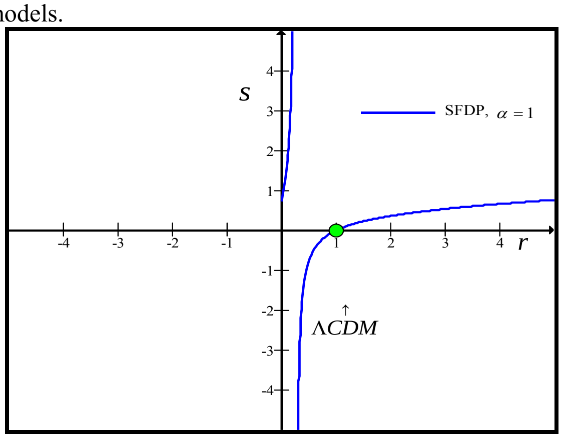

Fig.4. Statefinder parameters s v/s r.

One should hope that the future high precision observations will be capable of determining these statefinder parameters and consequently explore the nature of Dark Energy.

v) Coincidence problem: We know that the dark matter and dark energy must scale each other over a considerably long period of time during the later stage of evolution of the universe to avoid the coincidence problem. In other words, the ratio of two energy densities $\bar{r} = \rho_m / \rho_\Lambda$ remains constant in spite of their different rates of time evolution.

Here, it is interesting to note that the coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution in a special form of deceleration parameter cosmological model provided that $b^2 = 1$.

The variation of coincidence parameter $\bar{r}$ with respect to cosmic time *t* is as shown in figure-5.

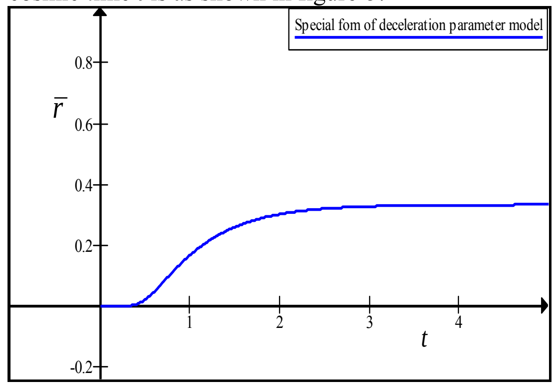

Fig.5. Coincidence parameter $\bar{r}$ verses time *t*.

Thus, in case of a plane symmetric cosmological model, the interaction between holographic dark energy and dark matter can make the ratio of their densities possible to attain a stationary value during the course of evolution and consequently can help alleviating the coincidence problem.

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