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On Estimations of Parameters of Generalized Gompertz Distribution

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Abstract **– In this paper, we discuss the maximum likelihood and Bayes estimators for the three parameters of Generalized Gompertz distribution based on Type-II censored samples. However, the Bayes estimators are studied under three different types of loss function; squared error, Linear-exponential and general entropy loss functions. Two methods are used for computing the Bayes estimates; standard Bayes and importance sampling. The performances of the estimates are compared through the mean square error by Monte Carlo simulation study.**

Keywords **– Exponentiated Gompertz Distribution, Squared Error Loss Function, LINEX Loss Function, General Entropy Loss Function.**

I. INTRODUCTION

The three-parameter Generalized Gompertz (GGpz) distribution has the distribution function is,

$$F(x) = \left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta},\ \theta, \lambda, \alpha, x > 0 \qquad (1)$$

This function was introduced via El-Gohary, Alshamrani and Al-Otaibi [10], when λ = δ/c, α = c and θ = θ. We will write X~ GGpz (θ, λ, α) to denote that the random variable X follows an Generalized Gompertz distribution with α is scale parameter and θ, λ are shape parameters. Therefore, the probability density function (pdf) is then given by

$$f(x) = \theta\lambda\alpha e^{\alpha x} e^{-\lambda(e^{\alpha x}-1)}\left(1 - e^{-\lambda(e^{\alpha x}-1)}\right)^{\theta-1} \qquad (2)$$

Several properties of this distribution discussed by Abu-Zinadah and Aloufi [4], some statistical measures of this distribution given, and they derive the quantiles, median and mode. They introduce the survival and hazard functions and determine the nature of the failure rate curve. They discussed mean residual life function, mean deviation and Rényi entropy. In addition, Abu-Zinadah [1], discussed the maximum likelihood estimator, method of moment estimator, estimator based on percentiles, least squares estimator and weighted least squares estimator of the shape parameter θ of EGpz distribution. Also, Abu-Zinadah [2], derive the Bayes estimates of parameter θ of EGpz distribution. Moreover, Abu-Zinadah [3], discussed goodness-of-fit tests for testing the three parameters EGpz distribution.

In Bayes estimator use the three loss functions. One is the squared error loss function that is categorized as a symmetric function and associates equal importance to the losses for overestimation and underestimation of equal magnitude. Others are LINEX and general entropy loss functions that are asymmetric. The LINEX loss function rise approximately exponentially on one side of zero and approximately linearly on the other side was presented by Varian [24]. This function was usually used by some authors; for example [5], [6], [14]–[17], [19], [20], [25]. The general entropy (GE) loss is also asymmetric loss function that is used by some authors, among of them, [8], [9], [18], [21], [22].

The goal of this article is to derive the maximum likelihood (ML) and Bayesian estimation under squared error (SE), Linear-exponential (LINEX) and general entropy (GE) loss functions based on censored samples Type-II.

In Bayes estimation, we recommended two methods, standard Bayes technique and importance sampling technique. Importance sampling method is a prevalent sampling tool used for Monte Carlo computing. It is used for numerically approximating integrals. Also, it is observed as a difference reduction technique. In addition, this method is used in an assortment of applications (see [7], [11]–[13], [23]).

The article is organized into five sections. In Section 2, we discuss the ML estimation for three parameters. The Bayesian estimation under SE, LINEX and GE loss functions are reviewed in Section 3 by using two methods for computing the Bayes estimates. The performances of the estimates are compared through the mean squared errors (MSE's) by using Monte Carlo simulation study based on different sample size are implementing in section 4. Finally, some concluding remarks are discussed in Section 5.

II. MAXIMUM LIKELIHOOD ESTIMATION

Assume that is $\underline{x} = (x_1, x_2, \dots, x_r)$ with $x_1 < x_2 < \dots < x_r$ is a censored sample Type-II of size r is obtained from a life testing on n items whose lifetimes have the GGpz (θ, λ, α) model. It is supposed that parameters θ, λ and α are unknown, the likelihood function for parameters θ, λ and α as,

$$l(x; \theta, \lambda, \alpha) = \frac{n!}{(n-r)!} \prod_{i=1}^{r} f(x_i)\, [1 - F(x_0)]^{n-r} \qquad (3)$$

Where f (x) and F (x) are the density and distribution functions respectively, and in Type-II the time of termination at $x_0 = x_r$. When they θ, λ and α are unknown then the likelihood function (3) with distribution function (1) and probability density function (2) is

$$l(x;\theta,\lambda,\alpha) = \frac{n!}{(n-r)!} \times (\theta\lambda\alpha)^r e^{\alpha\sum_{i=1}^{r} x_i} e^{-\lambda\sum_{i=1}^{r}(e^{\alpha x_i}-1)} \prod_{i=1}^{r}[1-$$
e−λeαxi−1]θ−1[1−F(xr;θ,λ,α)]n−r (4)

Then the log-likelihood function (4) given,

$$L = L(x;\theta,\lambda,\alpha) = ln\left(\frac{n!}{(n-r)!}\right) + r\ ln(\theta\lambda\alpha) + \alpha\sum_{i=1}^{r} x_i - \lambda\sum_{i=1}^{r}(e^{\alpha x_i}-1) + (\theta-1)\sum_{i=1}^{r} ln\left(1-e^{-\lambda(e^{\alpha x_i}-1)}\right) + (n-r)\, ln[1-\ F(x_r;\theta,\lambda,\alpha)] \quad (5)$$

Thus, the ML estimations of θ, λ and α, says $\hat{\theta}_{ML}, \hat{\lambda}_{ML}$ and $\hat{\alpha}_{ML}$, can be obtaining by maximize (5) directly by respect to θ, λ and α or we can solve the following non-linear equations using iterative procedure:

$$\frac{\partial L}{\partial\theta} = 0 \Longrightarrow \frac{r}{\theta} + \sum_{i=1}^{r} ln\left[1-e^{-\lambda(e^{\alpha x_i}-1)}\right] - (n-$$
r∂∂θFxr1− Fxr=0 (6)

$$\frac{\partial L}{\partial\lambda} = 0 \Longrightarrow \frac{r}{\lambda} + \sum_{i=1}^{n}(e^{\alpha x_i}-1)\left[(\theta-1)\frac{e^{-\lambda(e^{\hat{\alpha} x_i}-1)}}{1-e^{-\lambda(e^{\alpha x_i}-1)}} - \right.$$
1−n−r∂∂λFxr1− Fxr=0 (7)

$$\frac{\partial L}{\partial\alpha} = 0 \Longrightarrow \frac{r}{\alpha} + \sum_{i=1}^{r} x_i + \lambda\sum_{i=1}^{r} x_i\, e^{\alpha x_i}\left[(\theta-1)\frac{e^{-\lambda(e^{\alpha x_i}-1)}}{1-e^{-\lambda(e^{\alpha x_i}-1)}} - 1\right] - (n-r)\frac{\frac{\partial}{\partial\alpha}F(x_r)}{[1-F(x_r)]} = 0 \quad (8)$$

Where,

$$\frac{\partial}{\partial\theta}\ F(x_r) = \left[1-e^{-\lambda\left(e^{-\lambda(e^{\alpha x_r}-1)}\right)}\right]^{\theta} ln[1-$$
e−λeαxr−1

$$\frac{\partial}{\partial\lambda}\ F(x_r) = \theta(e^{\alpha x_r}-1)e^{-\lambda(e^{\alpha x_r}-1)}[1-$$
e−λeαxr−1θ−1

$$\frac{\partial}{\partial\alpha}\ F(x_r) = \theta\lambda x_r e^{\alpha x_r} e^{-\lambda(e^{\alpha x_r}-1)}[1-$$
e−λeαxr−1θ−1

Note that, if r = n the equations in (6), (7) and (8) reduce to the normal equations from complete sample. The system of the non-linear equations (6), (7) and (8) solve by numerical techniques and mathematical packages.

III. Bayes Estimation

In this section, we assuming the parameters (θ, λ, α) are unknown, let the prior of θ is gamma (a, b), prior of λ is gamma (c, d) and prior of α is gamma (e, f),

$$\pi(\theta) = \frac{b^a}{\Gamma(a)}\ \theta^{a-1}e^{-b\theta}\ . \quad (9)$$

$$\pi(\lambda) = \frac{d^c}{\Gamma(c)}\ \lambda^{c-1}e^{-d\lambda}\ . \quad (10)$$

and

$$\pi(\alpha) = \frac{f^e}{\Gamma(e)}\ \alpha^{e-1}e^{-f\alpha}\ . \quad (11)$$

Then the joint prior is

$$\pi(\theta,\lambda,\alpha) = \frac{b^a}{\Gamma(a)}\ \theta^{a-1}e^{-b\theta} \times \frac{d^c}{\Gamma(c)}\ \lambda^{c-1}e^{-d\lambda} \times \frac{f^e}{\Gamma(e)}\ \alpha^{e-1}e^{-f\alpha} \quad (12)$$

Where all the hyperparameters a, b, c, d, e, f are supposed to be known and non-negative.

Then, combining (4) and (12), the joint posterior density function of θ, λ and α given the x can be written as

$$P(\theta,\lambda,\alpha|x) = k\theta^{r+a-1}\lambda^{r+c-1}\alpha^{r+e-1}\ exp\left\{-\left[\theta(b-\right.\right.$$
i=1rln1−e−λeαxi−1+λd+i=1reαxi−1+αf−i=1rxi−n−rln1−1−e−λeαxr−1θ−i=1rln1−e−λe αxi−1 (13)

Where,

$$k^{-1} = \iiint_0^{\infty} \theta^{r+a-1}\lambda^{r+c-1}\alpha^{r+e-1}\ exp\left\{-\left[\theta(b-\right.\right.$$
i=1rln1−e−λeαxi−1+λd+i=1reαxi−1+αf−i=1rxi −n−rln1−1−e−λeαxr−1θ−i=1rln1−e−λeαxi−1dα dλdθ. (14)

The Bayes estimators of function of θ, λ and α, say $\varphi\ (\theta,\lambda,\alpha)$. These are obtained under three different types of loss functions as follows:

1. Squared error loss function

The Bayes estimator of $\varphi\ (\theta,\lambda,\alpha)$, under squared error (SE) loss function, denoted by $\hat{\varphi}_{BS}(\theta,\lambda,\alpha)$ be obtained as,

$$\hat{\varphi}_{BS}(\theta,\lambda,\alpha) = k\ \iiint_0^{\infty} \varphi(\theta,\lambda,\alpha)\theta^{r+a-1}\lambda^{r+c-1}\alpha^{r+e-1} \times$$
exp−θb−i=1rln1−e−λeαxi−1+λd+i=1reαxi−1+α f−i=1rxi−n−rln1−1−e−λeαxr−1θ−i=1rln1−e−λ eαxi−1dαdλdθ (15)

2. LINEX loss function

Under a LINEX loss function, the Bayes estimator of $\varphi\ (\theta,\lambda,\alpha)$ denoted by $\hat{\varphi}_{BL}(\theta,\lambda,\alpha)$ can be obtained as,

$$\hat{\varphi}_{BL}(\theta,\lambda,\alpha) = -\frac{1}{h}ln\left[k\ \iiint_0^{\infty} e^{-h\varphi(\theta,\lambda,\alpha)}\ \theta^{r+a-1}\lambda^{r+c-1}\alpha^{r+e-1} \times\right.$$
exp−θb−i=1rln1−e−λeαxi−1+λd+i=1reαxi−1+α f−i=1rxi−n−rln1−1−e−λeαxr−1θ −i=1rln1−e−λeαxi−1dαdλdθ. (16)

3. General entropy loss function

Under a general entropy (GE) loss function, the Bayes estimator of $\varphi\ (\theta,\lambda,\alpha)$, denoted by $\hat{\varphi}_{BG}(\theta,\lambda,\alpha)$ can be obtained as

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$\hat{\varphi}_{BG}(\theta,\lambda,\alpha) =$
$\Big[k \iiint_0^{\infty} [\varphi(\theta,\lambda,\alpha)]^{-q} \theta^{r+a-1} \lambda^{r+c-1} \alpha^{r+e-1} \, exp\Big\{-[\theta(b -$
i=1rln[1−e−λeaxi−1])+λd+i=1reaxi−1+af−i=1
rxi−n−rln1−1−e−λeaxr−1θ−i=1rln1−e−λeaxi−1
]dadλdθ−1q. (17)

Where k^{-1} is defined in (14). It is not possible to calculate (15), (16) and (17) analytically.

Hence, we suggested using a numerical integration technique using two methods; standard Bayes and importance sampling techniques.

3.1 Standard Bayes technique

For given value of hyperparameters a, b, c, d, e and f, Bayes estimates under SE, LINEX and GE loss functions are calculated using a mathematical integration to solve (15), (16) and (17).

3.2 Importance sampling technique

The importance sampling method is proposed to calculate the Bayes estimates. The joint posterior density function (13) can be written a

$P(\theta,\lambda,\alpha|\underline{x}) \propto$
$\frac{\left(b-\sum_{i=1}^{r} ln\left[1-e^{-\lambda(e^{\alpha x_i}-1)}\right]\right)^{r+a}}{\Gamma(r+a)} \theta^{r+a-1} \, exp\{-\theta(b -$
i=1rln1−e−λeaxi−1 ×d+i=1reaxi−1r+cΓr+c
$\lambda^{r+c-1} \, exp\{-\lambda(d + \sum_{i=1}^{r}(e^{\alpha x_i} - 1))\} \times$
$\frac{(f-\sum_{i=1}^{r} x_i)^{r+e}}{\Gamma(r+e)} \alpha^{r+e-1} \, exp\{-\alpha(f - \sum_{i=1}^{r} x_i)\} \times$
$\frac{exp\left\{(n-r)\, ln\left(1-\left[1-e^{-\lambda(e^{\alpha x_r}-1)}\right]^{\theta}\right)-\sum_{i=1}^{r} ln\left[1-e^{-\lambda(e^{\alpha x_i}-1)}\right]\right\}}{\left(b-\sum_{i=1}^{r} ln\left[1-e^{-\lambda(e^{\alpha x_i}-1)}\right]\right)^{r+a}\left(d+\sum_{i=1}^{r}(e^{\alpha x_i}-1)\right)^{r+c}};$

Then, the joint posterior density functions for θ, λ and α can be considered as

$P(\theta,\lambda,\alpha|\underline{x}) \propto \; Gamma(\theta; r+a\,, b - \sum_{i=1}^{r} ln[1 -$
e−λeaxi−1× Gammaλ;r+c,
d+i=1reaxi−1× Gammaα;r+e, (18)

Where,

$$g(\theta,\lambda,\alpha|x) = \frac{exp\left\{(n-r)\, ln\left(1-\left[1-e^{-\lambda(e^{\alpha x_r}-1)}\right]^{\theta}\right)-\sum_{i=1}^{r} ln\left[1-e^{-\lambda(e^{\alpha x_i}-1)}\right]\right\}}{\left(b-\sum_{i=1}^{r} ln\left[1-e^{-\lambda(e^{\alpha x_i}-1)}\right]\right)^{r+a}\left(d+\sum_{i=1}^{r}(e^{\alpha x_i}-1)\right)^{r+c}} \quad (19)$$

It is probable to use the importance sampling technique to calculate the Bayes estimates of any $\varphi\,(\theta,\lambda,\alpha)$ using (18). The right-hand side of (18), say $P_N(\theta,\lambda,\alpha|\underline{x})$, and $P\,(\theta,\lambda,\alpha|\underline{x})$ differ only by the proportionality constant.

The approximate Bayes estimator of $\varphi\,(\theta,\lambda,\alpha)$ under the SE, LINEX and GE loss functions says $\hat{\varphi}(\theta,\lambda,\alpha)_{SBS}$, $\hat{\varphi}(\theta,\lambda,\alpha)_{SBL}$ and $\hat{\varphi}(\theta,\lambda,\alpha)_{SBG}$, respectively. On to Kundu and Howlader [12] and Kundu and Pradhan [13], the next Algorithm by importance sampling method is as:

Step1. Generate $\alpha_1 \sim Gamma\,(r+e, f - \sum_{i=1}^{r} x_i)$,
$\lambda_1|\alpha_1 \sim$ Gamma $(r+c\,, d + \sum_{i=1}^{r}(e^{\alpha_1 x_i} - 1))$ and
$\theta_1|\lambda_1,\alpha_1 \sim Gamma\big(r+a\,, b - \sum_{i=1}^{r} ln\big[1 - e^{-\lambda_1(e^{\alpha_1 x_i}-1)}\big]\big).$

Step2. Repeat step 1 to obtain $(\theta_1,\lambda_1,\alpha_1), (\theta_2,\lambda_2,\alpha_2), \ldots, (\theta_N,\lambda_N,\alpha_N)$.

Step3. Compute the value

$$\hat{\varphi}(\theta,\lambda,\alpha)_{SBS} = \frac{\sum_{\tau=1}^{N} \varphi(\theta_\tau,\lambda_\tau,\alpha_\tau) g(\theta_\tau,\lambda_\tau,\alpha_\tau|x)}{\sum_{\tau=1}^{N} g(\theta_\tau,\lambda_\tau,\alpha_\tau|x)} \quad (20)$$

$$\hat{\varphi}(\theta,\lambda,\alpha)_{SBL} = -\frac{1}{h} ln\left[\frac{\sum_{\tau=1}^{N} e^{-h\varphi(\theta_\tau,\lambda_\tau,\alpha_\tau)} g(\theta_\tau,\lambda_\tau,\alpha_\tau|x)}{\sum_{\tau=1}^{N} g(\theta_\tau,\lambda_\tau,\alpha_\tau|x)}\right] \quad (21)$$

$$\hat{\varphi}(\theta,\lambda,\alpha)_{SBG} = \left[\frac{\sum_{\tau=1}^{N} [\varphi(\theta_\tau,\lambda_\tau,\alpha_\tau)]^{-q} g(\theta_\tau,\lambda_\tau,\alpha_\tau|x)}{\sum_{\tau=1}^{N} g(\theta_\tau,\lambda_\tau,\alpha_\tau|x)}\right]^{\frac{-1}{q}} \quad (22)$$

Where, $g\,(\theta_\tau,\lambda_\tau,\alpha_\tau|x)$ is defined in (19)

IV. Simulation Study

In this section, we discuss the numerical outcomes of a simulation study is performed to comparing the methods of ML and Bayes estimators, under SE, LINEX and GE loss functions for three-parameter of the GGpz (θ, λ, α) distribution. All computations are performed using Mathematica 9.0.

The next subsections explain the steps for obtaining ML and Bayes estimators for three-parameter numerically.

4.1 Maximum Likelihood Estimators

The ML estimators for θ, λ and α are obtained numerically according to the following steps.

Step1. For given values of prior parameters a, b, c, d, e and f generate a random values for θ, λ and α from the gamma distribution the density functions given in equations (9), (10) and (11), respectively.

Step2. Using θ, λ and α obtained in step 1 (θ =2.13668, λ =1.78059 and α =0.2968), to generating random samples of sizes n from the generation random variables $X = \frac{1}{\alpha} ln[1 - \frac{1}{\lambda} ln(1 - U^{\frac{1}{\theta}})]$ where X is GGpz (θ, λ, α) and U is a $uniform$ (0,1) distribution.

Step3. The ML estimator of the parameters θ, λ and α are computed by solving nonlinear equations (6), (7) and (8), respectively.

4.2 Standard Bayes Estimators.

The standard Bayes estimators for θ, λ and α are obtained numerically by accomplishment the preceding Steps 1 and 2 and then conducting the following step 3.

Step3. For given values of prior parameters (a=3,b=3,c=3,d=2,e =1.5,f = 0.5), we have taken the prior means to be same as the original means. The Bayes estimates of the parameters θ, λ and α, under SE, LINEX and GE loss functions are calculated by solving equations (15), (16) and (17), respectively.

4.3 Importance Sampling Estimators

The importance sampling technique of Bayes estimators for θ, λ and α are obtained numerically by accomplishment the preceding Steps 1 and 2 in subsection 4.1 and then conducting the following step 3.

Step3. For given values of (a =30, b =13, c= 23, d =10, e = 10, f = 346), the Bayes estimates of the parameters θ, λ and α, under SE, LINEX and GE loss functions are calculated by solving equations (20), (21) and (22), respectively.

All above Steps for the estimations of parameters θ, λ and α using ML and both techniques for Bayes estimators are refined 1000 times to calculate the mean square error (MSE). The simulations were carried out for censoring percentages of 70%, 80%, 90% and 100%, for each sample size n from GGpz distribution. Three different values of the LINEX shape parameters (h= -2, 0.001 and -2), and three different values of the GE shape parameters (q= -2, -1 and 2), are considered. In the importance sampling technique the results are obtained by using importance samples of size N=1000. The computational results are shown in (Table I) and (Table II).

Table I: Estimators and MSEs of ML and Bayesian estimates for the three parameters using standard Bayes method.

					LINEX			GE		
n	r	Par	ML	SE	$h=-2$	$h=0.001$	$h=2$	$q=-2$	$q=-1$	$q=2$
10	7	$\hat{\alpha}$	0.41653 (0.14802)	0.49326 (0.05007)	0.53761 (0.07663)	0.49325 (0.05007)	0.46211 (0.03619)	0.53059 (0.06979)	0.49326 (0.05007)	0.40065 (0.01652)
		$\hat{\lambda}$	1.9452 (2.11145)	0.94299 (0.71009)	1.45173 (0.15945)	0.94286 (0.71031)	0.74154 (1.08369)	1.09256 (0.48411)	0.94299 (0.71009)	0.57899 (1.44698)
		$\hat{\theta}$	2.44606 (1.81341)	1.56654 (0.39398)	2.00831 (0.19931)	1.5664 (0.39413)	1.33528 (0.67783)	1.64969 (0.30674)	1.56654 (0.39398)	1.30141 (0.74191)
	8	$\hat{\alpha}$	0.39728 (0.11064)	0.484722 (0.04637)	0.51622 (0.06262)	0.484711 (0.04636)	0.45165 (0.03159)	0.51207 (0.05863)	0.484722 (0.04637)	0.395839 (0.01482)
		$\hat{\lambda}$	1.88437 (1.9566)	0.96118 (0.68199)	1.45317 (0.15649)	0.961032 (0.68222)	0.75019 (1.06721)	1.10204 (0.47278)	0.96118 (0.68198)	0.58797 (1.42671)
		$\hat{\theta}$	2.46299 (1.88808)	1.5488 (0.41389)	1.97067 (0.20188)	1.54866 (0.41403)	1.32872 (0.68988)	1.63436 (0.32227)	1.5488 (0.41389)	1.29528 (0.75389)
	9	$\hat{\alpha}$	0.37129 (0.09193)	0.47240 (0.03956)	0.49814 (0.05289)	0.47238 (0.03956)	0.44609 (0.03008)	0.495991 (0.05044)	0.47240 (0.03956)	0.39501 (0.01501)
		$\hat{\lambda}$	1.84635 (1.92951)	0.97160 (0.66909)	1.45918 (0.15505)	0.97143 (0.66936)	0.76496 (1.03911)	1.11272 (0.46178)	0.97160 (0.66909)	0.60316 (1.39271)
		$\hat{\theta}$	2.27622 (1.70197)	1.54926 (0.41389)	1.93722 (0.22367)	1.54908 (0.41408)	1.3231 (0.70013)	1.61829 (0.34435)	1.54926 (0.41389)	1.29002 (0.76398)
	10	$\hat{\alpha}$	0.40695 (0.14048)	0.453959 (0.03369)	0.47982 (0.04465)	0.453959 (0.03368)	0.43637 (0.02679)	0.47889 (0.04325)	0.453979 (0.03369)	0.39103 (0.01412)
		$\hat{\lambda}$	2.09508 (2.65549)	0.996332 (0.63485)	1.47731 (0.14752)	0.99618 (0.63485)	0.78209 (1.00862)	1.13217 (0.44186)	0.996332 (0.63462)	0.62278 (1.35079)
		$\hat{\theta}$	2.55236 (1.81333)	1.51561 (0.45138)	1.91339 (0.22931)	1.51548 (0.45153)	1.32335 (0.69861)	1.60701 (0.35662)	1.51561 (0.45138)	1.29085 (0.76128)
30	21	$\hat{\alpha}$	0.35543 (0.05153)	0.47286 (0.03761)	0.50094 (0.04900)	0.47284 (0.03759)	0.44878 (0.02873)	0.49946 (0.04768)	0.47286 (0.03761)	0.39907 (0.01465)
		$\hat{\lambda}$	1.93008 (1.5044)	0.97744 (0.65986)	1.40786 (0.17058)	0.97729 (0.66009)	0.77429 (1.02077)	1.11259 (0.46102)	0.97744 (0.65986)	0.61332 (1.36942)
		$\hat{\theta}$	2.31459 (0.76467)	1.68804 (0.26791)	1.96099 (0.17281)	1.68796 (0.26797)	1.52746 (0.41835)	1.75994 (0.22086)	1.68804 (0.26791)	1.53038 (0.42493)
	24	$\hat{\alpha}$	0.32855 (0.03692)	0.460326 (0.03244)	0.481558 (0.04051)	0.460336 (0.03244)	0.440462 (0.02538)	0.481274 (0.03987)	0.460326 (0.03244)	0.395749 (0.01345)
		$\hat{\lambda}$	1.99187 (1.49207)	0.999384 (0.62987)	1.43337 (0.15842)	0.99926 (0.63006)	0.784285 (1.00415)	1.13747 (0.4338)	0.999384 (0.62987)	0.626241 (1.3425)
		$\hat{\theta}$	2.28186 (0.74174)	1.68194 (0.27283)	1.92722 (0.16712)	1.68184 (0.27292)	1.53291 (0.40428)	1.73976 (0.22712)	1.68194 (0.27283)	1.53584 (0.40887)
	27	$\hat{\alpha}$	0.32265 (0.03599)	0.44934 (0.02839)	0.46765 (0.03564)	0.44933 (0.02839)	0.42960 (0.02197)	0.46803 (0.03534)	0.44934 (0.02839)	0.39015 (0.01209)
		$\hat{\lambda}$	1.98865 (1.70145)	1.0168 (0.61142)	1.44986 (0.15878)	1.01665 (0.61163)	0.81381 (0.95000)	1.1548 (0.42026)	1.0168 (0.61142)	0.65966 (1.27122)
		$\hat{\theta}$	2.17928 (0.74922)	1.67757 (0.27124)	1.92448 (0.16549)	1.67748 (0.27131)	1.52855 (0.40871)	1.74066 (0.22528)	1.67757 (0.27124)	1.53114 (0.41311)
	30	$\hat{\alpha}$	0.32827 (0.02401)	0.42593 (0.02156)	0.444411 (0.02816)	0.42595 (0.02157)	0.415579 (0.01891)	0.445443 (0.02816)	0.42593 (0.02156)	0.382802 (0.01127)
		$\hat{\lambda}$	2.1175 (1.67852)	1.06667 (0.55181)	1.47926 (0.16334)	1.06655 (0.55197)	0.852319 (0.89029)	1.19075 (0.39461)	1.06667 (0.55181)	0.710922 (1.17488)
		$\hat{\theta}$	2.37725 (0.71015)	1.65884 (0.28624)	1.89965 (0.16734)	1.65876 (0.28629)	1.50996 (0.43069)	1.72402 (0.23452)	1.65884 (0.28624)	1.51078 (0.43679)
50	35	$\hat{\alpha}$	0.33049 (0.02609)	0.46556 (0.03369)	0.48968 (0.04351)	0.46559 (0.03371)	0.44529 (0.02617)	0.48907 (0.04275)	0.46556 (0.03369)	0.3995 (0.01373)
		$\hat{\lambda}$	2.00184 (1.32163)	1.00446 (0.62344)	1.42791 (0.16289)	1.00437 (0.62362)	0.78845 (0.99576)	1.14062 (0.4318)	1.00446 (0.62344)	0.63067 (1.33244)
		$\hat{\theta}$	2.28209 (0.39336)	1.74233 (0.21625)	1.93669 (0.14235)	1.74375 (0.21776)	1.60682 (0.32025)	1.79063 (0.18455)	1.74233 (0.21625)	1.61584 (0.31769)
	40	$\hat{\alpha}$	0.32925 (0.02561)	0.450025 (0.02822)	0.467753 (0.03448)	0.44991 (0.02818)	0.43454 (0.02326)	0.468172 (0.03461)	0.450025 (0.02822)	0.394197 (0.01294)
		$\hat{\lambda}$	1.99503 (1.37804)	1.03726 (0.58244)	1.46222 (0.15068)	1.03707 (0.58274)	0.817085 (0.94613)	1.17468 (0.39707)	1.03726 (0.58244)	0.664481 (1.26282)
		$\hat{\theta}$	2.2744 (0.42859)	1.73192 (0.21878)	1.91506 (0.14271)	1.73185 (0.21882)	1.60498 (0.31786)	1.77652 (0.18979)	1.73192 (0.21878)	1.61347 (0.31491)

	45	$\hat{\alpha}$	0.31375 (0.02055)	0.43305 (0.02299)	0.44843 (0.02781)	0.43304 (0.02299)	0.42023 (0.01943)	0.44949 (0.02787)	0.43305 (0.02299)	0.38569 (0.01133)
		$\hat{\lambda}$	2.00023 (1.49961)	1.0743 (0.53813)	1.48691 (0.14356)	1.0743 (0.53823)	0.86132 (0.87094)	1.20263 (0.37149)	1.07435 (0.53813)	0.71769 (1.15659)
		$\hat{\theta}$	2.17754 (0.41496)	1.74633 (0.20367)	1.89973 (0.13814)	1.74622 (0.20375)	1.60164 (0.32149)	1.76676 (0.19014)	1.74633 (0.20367)	1.60981 (0.31878)
	50	$\hat{\alpha}$	0.31253 (0.01349)	0.41135 (0.017601)	0.424615 (0.021597)	0.41135 (0.017601)	0.40076 (0.014757)	0.426024 (0.021755)	0.41135 (0.017602)	0.373498 (0.009156)
		$\hat{\lambda}$	2.09421 (1.41411)	1.14135 (0.470253)	1.52026 (0.162476)	1.1412 (0.470253)	1.13677 (0.772886)	1.24791 (0.348995)	1.14135 (0.470076)	0.801803 (1.00603)
		$\hat{\theta}$	2.28062 (0.40389)	1.74512 (0.203075)	1.9065 (0.13319)	1.74507 (0.203108)	1.74301 (0.313364)	1.77478 (0.184169)	1.74512 (0.203075)	1.6222 (0.310592)

Table II: Estimators and MSEs of ML and Bayesian estimates for three parameters using importance sampling method.

					LINEX			GE		
n	r	Par	ML	SE	$h=-2$	$h=0.001$	$h=2$	$q=-2$	$q=-1$	$q=2$
10	7	$\hat{\alpha}$	0.43473 (0.1602)	0.06467 (0.05389)	0.06489 (0.05379)	0.06467 (0.05389)	0.06446 (0.05399)	0.06632 (0.05314)	0.06467 (0.05389)	0.05949 (0.05633)
		$\hat{\lambda}$	1.91154 (2.1717)	3.05281 (1.6219)	3.37715 (2.55894)	3.05267 (1.62155)	2.79196 (1.0250)	3.10008 (1.74447)	3.05281 (1.6219)	2.90839 (1.27471)
		$\hat{\theta}$	2.49459 (2.0114)	1.63656 (0.26861)	1.72614 (0.19209)	1.63652 (0.26864)	1.55788 (0.34989)	1.66187 (0.24465)	1.63656 (0.26861)	1.56007 (0.34891)
	8	$\hat{\alpha}$	0.36619 (0.0934)	0.07218 (0.05048)	0.07243 (0.05037)	0.07218 (0.05048)	0.07194 (0.05059)	0.07384 (0.04974)	0.07218 (0.05048)	0.06688 (0.05288)
		$\hat{\lambda}$	1.98003 (2.0711)	3.11966 (1.79953)	3.44674 (2.79312)	3.11951 (1.79913)	2.85674 (1.16198)	3.16635 (1.92736)	3.11966 (1.79953)	2.97737 (1.43746)
		$\hat{\theta}$	2.42703 (1.8257)	1.66494 (0.24482)	1.75771 (0.17214)	1.66489 (0.24486)	1.5833 (0.32423)	1.69073 (0.22198)	1.66494 (0.24482)	1.5868 (0.32234)
	9	$\hat{\alpha}$	0.37149 (1.8434)	0.08179 (0.04628)	0.08205 (0.04616)	0.08179 (0.04628)	0.08152 (0.04639)	0.08337 (0.04560)	0.08179 (0.04628)	0.07653 (0.04856)
		$\hat{\lambda}$	1.84339 (1.9482)	3.1563 (1.90648)	3.46073 (2.84779)	3.15616 (1.90609)	2.9004 (1.26285)	3.20057 (2.03112)	3.5163 (1.90648)	3.02043 (1.54896)
		$\hat{\theta}$	2.30624 (1.7763)	1.72197 (0.20021)	1.8156 (0.13788)	1.72193 (0.20024)	1.63704 (0.27239)	1.74757 (0.18052)	1.72197 (0.20021)	1.64329 (0.26882)
	10	$\hat{\alpha}$	0.41655 (0.1189)	0.09222 (0.04191)	0.09246 (0.04181)	0.09222 (0.04191)	0.09197 (0.04201)	0.09352 (0.04138)	0.09222 (0.04191)	0.08765 (0.04379)
		$\hat{\lambda}$	2.07005 (2.5077)	3.16403 (1.94195)	3.45602 (2.85304)	3.1639 (1.94157)	2.92043 (1.31834)	3.20612 (2.06147)	3.16403 (1.94195)	3.03561 (1.59962)
		$\hat{\theta}$	2.53018 (1.7048)	1.77886 (0.16035)	1.8714 (0.10986)	1.77882 (0.16037)	1.69331 (0.22280)	1.80365 (0.14417)	1.77886 (0.16035)	1.70223 (0.21804)
30	21	$\hat{\alpha}$	0.37215 (0.0693)	0.14639 (0.02276)	0.14665 (0.02268)	0.14639 (0.02276)	0.14613 (0.02283)	0.14727 (0.02249)	0.14639 (0.02276)	0.143303 (0.02369)
		$\hat{\lambda}$	1.88906 (1.4697)	3.13731 (1.90251)	3.37323 (2.61162)	3.13721 (1.90222)	2.9511 (1.42162)	3.1704 (1.99407)	3.13731 (1.90251)	3.03947 (1.64412)
		$\hat{\theta}$	2.26863 (0.7396)	1.91471 (0.08619)	1.97473 (0.06681)	1.91468 (0.08621)	1.8508 (0.11373)	1.93082 (0.07981)	1.91471 (0.08619)	1.8625 (0.11007)
	24	$\hat{\alpha}$	0.32382 (0.0321)	0.17094 (0.01598)	0.17109 (0.01595)	0.17094 (0.01598)	0.17078 (0.01602)	0.17140 (0.01587)	0.17094 (0.01598)	0.16933 (0.01639)
		$\hat{\lambda}$	2.02433 (1.5621)	2.92721 (1.39987)	3.10198 (1.84743)	2.92714 (1.39969)	2.79258 (1.09624)	2.9529 (1.46056)	2.92721 (1.39987)	2.85268 (1.23084)
		$\hat{\theta}$	2.30974 (0.7730)	1.98612 (0.06947)	2.03978 (0.05931)	1.98609 (0.06947)	1.93047 (0.08512)	1.99982 (0.06585)	1.98612 (0.06947)	1.94252 (0.08312)
	27	$\hat{\alpha}$	0.31537 (0.0306)	0.19208 (0.01113)	0.19217 (0.01111)	0.19208 (0.01113)	0.19199 (0.01115)	0.19231 (0.01108)	0.19208 (0.01113)	0.19131 (0.01129)
		$\hat{\lambda}$	1.99235 (1.5201)	2.67592 (0.92063)	2.79516 (1.16231)	2.67587 (0.92053)	2.57861 (0.74254)	2.69578 (0.95773)	2.67592 (0.92063)	2.61822 (0.81625)
		$\hat{\theta}$	2.22428 (0.7399)	2.04457 (0.07240)	2.09544 (0.07003)	2.04454 (0.07240)	1.99204 (0.07944)	2.05719 (0.07083)	2.04457 (0.07240)	2.00485 (0.07919)
	30	$\hat{\alpha}$	0.32172 (0.0434)	0.21437 (0.0069)	0.214432 (0.00697)	0.21437 (0.00698)	0.21431 (0.00699)	0.21451 (0.00696)	0.21437 (0.00698)	0.21391 (0.00706)
		$\hat{\lambda}$	2.20727 (1.9129)	2.28518 (0.38469)	2.35677 (0.46887)	2.28514 (0.38466)	2.22705 (0.32036)	2.299 (0.39908)	2.28518 (0.38469)	2.24573 (0.34426)
		$\hat{\theta}$	2.3932 (0.7401)	2.03992 (0.07493)	2.07816 (0.07159)	2.03991 (0.07493)	2.0004 (0.08041)	2.04944 (0.07351)	2.03992 (0.07493)	2.01008 (0.08027)
50	35	$\hat{\alpha}$	0.33329 (0.02886)	0.22697 (0.00507)	0.22708 (0.00505)	0.22697 (0.00507)	0.22685 (0.00509)	0.22723 (0.00503)	0.22697 (0.00507)	0.2261 (0.00519)
		$\hat{\lambda}$	1.98828 (1.33851)	2.51228 (0.62923)	2.59798 (0.77262)	2.51225 (0.62917)	2.44475 (0.52485)	2.52705 (0.65212)	2.51228 (0.62923)	2.47039 (0.56582)
		$\hat{\theta}$	2.25989 (0.40945)	2.09573 (0.05621)	2.13896 (0.05701)	2.0957 (0.05621)	2.05147 (0.05832)	2.10613 (0.05571)	2.09573 (0.05621)	2.06323 (0.05883)

	40	$\hat{\alpha}$	0.31046 (0.01682)	0.26352 (0.00133)	0.26358 (0.00133)	0.26352 (0.00133)	0.26346 (0.00133)	0.26363 (0.00132)	0.26352 (0.00133)	0.26316 (0.00136)
		$\hat{\lambda}$	2.06405 (1.43402)	2.12447 (0.20777)	2.17193 (0.2499)	2.12445 (0.20775)	2.08468 (0.174817)	2.13438 (0.21556)	2.12447 (0.20777)	2.09614 (0.18603)
		$\hat{\theta}$	2.26241 (0.41137)	2.10637 (0.06273)	2.14176 (0.06413)	2.10635 (0.06273)	2.06991 (0.06202)	2.1148 (0.06257)	2.10637 (0.06273)	2.0801 (0.06347)
	45	$\hat{\alpha}$	0.31049 (0.01892)	0.30238 (0.000297)	0.30242 (0.000297)	0.30238 (0.000297)	0.30234 (0.000297)	0.30245 (0.000297)	0.30238 (0.000297)	0.30216 (0.000297)
		$\hat{\lambda}$	1.98404 (1.27061)	1.72194 (0.07664)	1.74822 (0.07787)	1.72193 (0.07664)	1.69826 (0.07646)	1.72902 (0.07644)	1.72194 (0.07664)	1.70134 (0.07769)
		$\hat{\theta}$	2.19852 (0.44704)	2.09503 (0.07122)	2.12417 (0.07071)	2.09501 (0.07122)	2.06653 (0.07268)	2.1019 (0.07066)	2.09503 (0.07122)	2.07416 (0.0732)
	50	$\hat{\alpha}$	0.32179 (0.01578)	0.35056 (0.00327)	0.3506 (0.00328)	0.35056 (0.00327)	0.35052 (0.00327)	0.35062 (0.00328)	0.35056 (0.00327)	0.35038 (0.00326)
		$\hat{\lambda}$	2.02273 (1.22056)	1.24227 (0.34051)	1.25412 (0.32976)	1.24227 (0.34051)	1.23167 (0.35037)	1.24661 (0.33627)	1.24227 (0.34051)	1.22994 (0.35274)
		$\hat{\theta}$	2.26617 (0.36958)	1.9759 (0.09222)	1.99589 (0.08735)	1.9759 (0.09222)	1.95648 (0.09749)	1.98088 (0.09070)	1.9759 (0.09222)	1.96091 (0.09697)

V. Conclusion

The three-parameter GGpz distribution is considered for obtaining ML and Bayesian estimations for parameters based on Type-II censored sample. For assumed values of the hyperparameters a, b, c, d, e and f generated values of $\alpha = 0.2968$, $\lambda = 1.78059$ and $\theta = 2.13668$, then generate type-II censored sample of size n from GGpz distribution.

The ML is obtained by using Mathematica 9.0. In addition, it is supposed that the parameters have three independent gamma priors. The Bayes estimators under SE, LINEX and GE loss functions are calculated through two methods; standard Bayes and importance sampling are obtained by using Mathematica 9.0. The performances of the estimates are conducted by using the MSE.

We studies Monte Carlo simulation were carried out for censoring percentages of 70%, 80%, 90% and 100% (complete sample), for each sample size. From results in (Table I) and (Table II), we observed the following.

1) The MSEs of the estimates are decreasing as the sample size is increasing.
2) Under LINEX loss function when the LINEX constant is close to zero, (h = 0.001) the MSEs of the Bayes estimates are similar to their corresponding MSEs of the Bayes estimates under squared error loss function.
3) Under GE loss function when q = -1, the MSEs of Bayes estimates are extremely similar to their corresponding MSEs of the Bayes estimates under squared error loss function.
4) The standard Bayes and importance sampling techniques are given resulting estimates extremely close to each other.
5) Most of the time, of standard Bayes technique, The Bayes estimates of λ and θ under LINEX with parameter h = -2, have smaller MSEs as compared with their other corresponding estimates. Also, the Bayes estimates of α under GE with parameter q = 2, have smaller MSEs as compared with their other corresponding estimates.
6) By using the importance sampling technique, usually the Bayes estimates of θ under LINEX with parameter h = -2, have smaller MSEs and the Bayes estimates of λ under LINEX with parameter h = 2, have smaller MSEs as compared with their other corresponding estimates. Moreover, the Bayes estimates of α under GE with parameter q = -2, have smaller MSEs as compared with their other corresponding estimates.
7) Usually the parameter θ is underestimates for all estimates accepting ML estimator is overestimate.
8) Most of time the parameter λ is overestimate for all estimates accepting Bayes estimators through standard Bayes technique and by using importance sampling technique at n = 50 is underestimates.
9) Most of time the parameter α is overestimate for all estimates accepting Bayes estimator by using importance sampling technique at n =10, 30 are underestimates.
10) The Bayes estimates generally have smaller MSEs compared with their corresponding ML estimates.

References

[1] H. H. Abu-Zinadah, “Six Method of Estimations for the Shape Parameter of Exponentiated Gompertz Distribution”. Applied Mathematical Sciences, 8(88), 2014a, pp. 4349-4359.

[2] H. H. Abu-Zinadah, “Bayesian Estimation on the Exponentiated Gompertz Distribution under Type II Censoring”. International Journal of Contemporary Mathematical Sciences, 9(11), 2014b, pp. 497-505.

[3] H. H. Abu-Zinadah, “Goodness-of-Fit Tests for the Exponentiated Gompertz Distribution”. International Journal of Innovation in Science and Mathematics. 2(4), 2014c, pp. 2347-9051

[4] H. H. Abu-Zinadah and A. S. Aloufi, “Some Characterizations of the Exponentiated Gompertz Distribution”. International Mathematical Forum, 9(30), 2014, pp. 1427-1439.

[5] W. Afify, “On estimation of the exponentiated Pareto distribution under different sample schemes”. Statistical Methodology, 7(2), 2010, pp. 77-83.

[6] R. A. Bakoban, “A study on mixture of exponential and exponentiated gamma distributions”. Advances and Applications in Statistical Sciences, 2(1), 2010, pp. 101-127.

[7] R. A. Bakoban, “Bayesian and non Bayesian estimation of erlang distribution under progressive censoring”. International Journal of Research and Reviews in Applied Sciences, 11(3), 2012, pp. 524-535.

[8] D. K. Dey, M. Ghosh, and C. Srinivasan, “Simultaneous estimation of parameters under entropy loss”. Journal of Statistical Planning and Inference, 25, 1987, pp. 347-363.

[9] D. K. Dey, and P. L. Liu, “On comparison of estimators in a generalized life model”. Microelectronics Reliability, 32(1), 1992, pp. 207-221.

[10] A. El-Gohary, A. Alshamrani and A. N. Al-Otaibi, "The generalized Gompertz distribution". Applied Mathematical Modelling, 37(1), 2013, pp. 13-24.

[11] H. S. Klakattawi, L. A. Baharith and G. R. Al-Dayian, "Bayesian and non-Bayesian estimations on the exponentiated modified Weibull distribution for progressive censored sample". Communications in Statistics-Simulation and Computation, 40(9), 2011, pp. 1291-1309.

[12] D. Kundu and H. Howlader, "Bayesian inference and prediction of the inverse Weibull distribution for Type-II censored data". Computational Statistics and Data Analysis, 54(6), 2010, pp. 1547-1558.

[13] D. Kundu and B. Pradhan, "Bayesian inference and life testing plans for generalized exponential distribution". Science in China Series A: Mathematics, 52(6), 2009, pp. 1373-1388.

[14] M. M. Nassar and F. H. Eissa, "Bayesian estimation for the exponentiated Weibull model". Communications in Statistics-Theory and Methods, 33(10), 2005, pp. 2343-2362.

[15] A. Shawky and R. Bakoban, "Bayesian and non-Bayesian estimations on the exponentiated gamma distribution". Applied Mathematical Sciences, 2(51), 2008, pp. 2521-2530.

[16] U. Singh, P. K. Gupta and S. Upadhyay, "Estimation of exponentiated Weibull shape parameters under LINEX loss function". Communications in Statistics-Simulation and Computation, 31(4), 2002, pp. 523-537.

[17] U. Singh, P. K. Gupta and S. Upadhyay, "Estimation of three-parameter exponentiated-Weibull distribution under type-II censoring". Journal of Statistical Planning and Inference, 134(2), 2005, pp. 350-372.

[18] S. K. Singh, U. Singh and D. Kumar, "Bayesian estimation of the exponentiated gamma parameter and reliability function under asymmetric loss function". REVSTAT–Statistical Journal, 9(3), 2011, pp. 247-260.

[19] A. A. Soliman, "Comparison of linex and quadratic bayes estimators foe the rayleigh distribution". Communications in Statistics-Theory and Methods, 29(1), 2000, 95-107.

[20] A. A. Soliman, "Reliability estimation in a generalized life-model with application to the Burr-XII". IEEE Transactions on Reliability, 51(3), 2002, pp. 337-343.

[21] A. A. Soliman, "Estimation of parameters of life from progressively censored data using Burr-XII model". IEEE Transactions on Reliability, 54(1), 2005, pp. 34-42.

[22] A. A. Soliman, "Estimators for the finite mixture of Rayleigh model based on progressively censored data". Communications in Statistics-Theory and Methods, 35(5), 2006, pp. 803-820.

[23] S. T. Tokdar, and R. E. Kass, "Importance sampling: a review". Wiley Interdisciplinary Reviews: Computational Statistics, 2(1), 2010, pp. 54-60.

[24] H. R. Varian, "A Bayesian approach to real estate assessment". North Holland, Amsterdam, 1975, pp. 195-208.

[25] M. Yarmohammadi and H. Pazira, "Classical and Bayesian estimations on the generalized exponential distribution using censored data". International Journal of Mathematical Analysis, 4(29), 2010, pp. 1417-1431.

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