

The Self Similarity Equivalence Relation Connecting Newton's Energy with Einstein's Energy and Dark Energy

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Dedicated to Lydia, Shareen and Sonya who are self similar in the utmost positive way.

Abstract – In this paper we utilize some deep mathematical connections between equivalence classes of equivalence relations and E-infinity theory quotient space. We start from the basic principles of self similarity which came to prominence in science with the advent of the modern theory of nonlinear dynamical systems, deterministic chaos and fractals. This fundamental logico-mathematical thread related to partially ordered sets is then utilized to show how the classical Newton's kinetic energy $E = \frac{1}{2} mv^2$ leads to Einstein's celebrated maximal energy equation $E = mc^2$ and how in turn this can be dissected into the ordinary energy density $E(O) = mc^2/22$ and the dark energy density $E(D) = mc^2(21/22)$ of the cosmos where m is the mass, v is the velocity and c is the speed of light.

Keywords– Einfinity, Einstein Self Similarity, Equivalence Relation, Fractal Scaling, Fractal Quantum Field, Golden Mean Scaling, Intermediate Asymptotic, Quantum Gravity, Random Cantor Sets, Scaling, Special Relativity, tHooft Regularization.

I. INTRODUCTION

In mathematics, it is well known that not only Cantorian fractals probabilistic equivalence relation (see Appendixes 1-4) can be based on similarities and we will show here how this can lead to surprising connections to physics via Penrose fractal tiling based self similar E-infinity quotient spacetime manifold [1]-[20]. Self similarity may be seen as trivially obvious as the number ten, being ten copies of unity and/or as quite complex as the Julia and Mandelbrot set of fractal dynamics [1]-[6]. Self similarity (see Appendixes one to three) is in the meantime a well known fundamental principle of modern nonlinear dynamics, theory of fractals (see Figs. 1 -3) as well as deterministic and quantum chaos in physics and cosmology [7]-[18]. More recently fractal-Cantorian spacetime and E-infinity theory demonstrated how self similarity is intimately and closely related to the renormalization procedures [19]-[25] of quantum field theory and 'tHooft's dimensional regularization [26]. In fact the said E-infinity Cantorian spacetime theory [26]-[34] is built almost entirely on the explicit self similarity of the expectation value of its Hausdorff dimension [10], [28].

$$D = 4 + \phi^3$$

$$4 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4 + \dots}}}$$
(1)

where $\phi = (\sqrt{5} - 2)/2$ [7-17]. It should therefore be expected as a matter of logical consistency that the equation derived from and in this spacetime will also be self similar or at a minimum self affine [23]-[45]. This is actually the main theme of the present paper [1]-[106] and at the end of a rather detailed discussion and analysis we will rediscover a fundamental fact, namely that most of the experimentally verified theories were always self similar in one way or another [106]. Looking for instance at fields as diverse as classical gravity [17], [24], [30], electric fields as well as magnetic fields we find that the basic inverse square law reigns supreme [46]-[48]. In fact and as will be analysed in substantial detail in the present work, Newton's kinetic energy $E = \frac{1}{2} mv^2$ where m is the mass and v is the velocity and probably the most famous formula in physics, with which we mean Einstein's mass-energy equation $E = mc^2$ where c is the speed of light (see Fig. 3) differ only formally in the scale and notation used and nothing much more than that (see Figs. 3 and 4). Remembering that energy, entropy and information [46] [49]-[57] are without a doubt some of the most fundamental and interrelated notions in physics, then it is natural that we utilize these afore mentioned self similarity to shed light on the major problem of the measurable ordinary energy density [58]-[63] and the dark energy density [60-63] of the cosmos which we cannot measure at present in any direct way [57]-[63]. As mentioned earlier on, this and various related aspects [60]-[74] is the subject of the present paper which although relatively short, has the character of a survey paper due to the large number of subjects and problems we address as well as the numerous theoretical and mathematical physics references included [1]-[107]. On the other hand we give limited space to the pure mathematical literature of equivalence relations which is never the less vital to an even deeper understanding of our method and results. This short coming could be helped by consulting Refs. [106]-[111] (see also Appendix 4).

II. THE POWER OF BEING VAGUE

The title of this section is not in praise of being sloppy in physics but simply a somewhat provocative invitation to take fuzzy logic [11], fuzzy sets [76]-[85] and consequently Cantor sets, fractals and transfinite set theory very seriously in quantum physics [70]-[106]. Nature is overwhelmingly fuzzy in an irreducible and fundamental way as reflected in the mathematics of even the most orthodox quantum mechanics theories, Hardy's quantum

entanglement included [54]. Consequently to force nature to be described mathematically in a sharp, crisp way is really forcing it to submit to an inaccurate mathematical formulation [11]. As an example of what we have in mind, let us count quantum particles. At the beginning there was SU(3) SU(2) and U(1) of the standard model [13], [28], [46]. The number of the generators of this combined Lie symmetry group corresponds to a dimension equal to $8 + 3 + 1 = 12$ and this number corresponds, as is well known, to eight gluons, three massive photons and one massless ordinary photon [47]. So are quantum particle isometries of the same Lie symmetry spaces corresponding to Lie symmetry groups – this may be a naïve question not worthy of a question mark. In fact nature is far stranger than fiction because fractal logic [83] teaches us that these 12 messenger particles are in reality 14 particles and have the fractal weight number of 11.708239325 [83]. Now regardless how we answer this naïve question and the puzzling fractal logical reality, we hasten to say that E8E8 super string theory starts with 496 quasi massless gauge bosons and not 12 and one has normally to show how these 496 isometries are reduced via symmetry breaking to our observed 12 gauge bosons of the standard model [28], [47]. In addition we know from bosonic string theory that spacetime is 26 dimensional and when we imagine 16 dimensions running in an opposite direction to these 26, then we find $26 - 16 = 10$ which corresponds to one type of a so called heterotic super string theory [28], [47] of which one may be based on E8E8 or on SO(32) which leads to [101]-[105].

$$|\text{SO}(32)| = (32)(32 - 1)/2 = 496, \quad (2)$$

as in the E8E8 where E8 is the largest exceptional Lie symmetry group [28], [55], [57]. Thus we could really think of internal space dimensions as particles resulting from symmetry breaking related to Noether's conservation laws and symmetry theorem and if we venture into even more "fuzzy" spacetime dimensions [41] such as $D = 11$ of Witten, $D = 5$ of Kaluza-Klein then even the $D = 4$ of Einstein could be seen as a kind of pre-particles [47]. But what good could this wild mixing of basically different notions and concepts bring about except a little more chaos added to a mess of other problems? We could answer this point by working out something specific, namely determining the density of ordinary energy, dark energy, dark matter energy and pure dark energy density of the cosmos. This is what we will do next [63]-[77].

The standard model contains 126 particle-like states when considering super symmetry and disregarding fractal logic counting [28], [46]. In the case of super gravity this contain 2 more, namely 128 which corresponds to half of the 256 Einstein-Riemann independent curvature tensor components as given by $(4)^4 = 256$ [48]. Without super symmetric partners this corresponds to 64 particles in 4D, which we can detect only if we perform experiments in 4D. However real experiments can be done only in 3D with time as a parameter [47].

Taking our previous discussion into account as well as realizing that all energies must ultimately be a scaling of Einstein's maximal energy $E = mc^2$, we are inclined to conclude that a good estimate of the ordinary, real

measurable energy density of the universe must be $E = mc^2$ scaled by the ratio of real space $D = 3$ to the space of the standard model plus gravity which is in the afore mentioned fuzzy meaning $D = 64$ so that one finds approximately [36]-[45].

$$\begin{aligned} E(O) &\approx \left(\frac{3}{64}\right)(mc^2) \\ &\approx 0.046875 mc^2 \\ &\approx 4.7\% \end{aligned} \quad (3)$$

of the total Einstein energy in reasonable agreement with the cosmic measurements [36]-[47]. To account for dark energy we have the self explanatory scaling $\lambda \equiv (64 - 3)/64 = 95.312\%$, again in fair agreement with measurements [37]-[45]. Finally to find the scaling associated with dark matter, we see that it is reasonable to consider the ratio of the total number to the 12 messenger particles of the standard model plus the Higgs and the graviton making them 14 particles corresponding to the dimension of $|G_2| = 14$ exceptional Lie group or $\dim \text{OSP}(1/4) = 14$ of orthosymplectic group [28], for which one finds

$$\begin{aligned} D(\text{dark matter}) &\approx \left(\frac{14}{64}\right)(mc^2) \\ &\approx 21.875\% \end{aligned} \quad (4)$$

yet again in fair agreement with the limit which cosmic measurements set on the dark matter energy density [36]-[47], [60].

III. A BLACK AND WHITE CANTOR UNIVERSE MADE OF TWO CANTOR SETS

Following the pictorial logic of the one dimensionally embedded iterative triadic Cantor set of Fig. 1 we see that it leads us to consider two sets (see also appendix 1). The first is the classical triadic set consisting of an uncountable infinite number of black Cantor points with a Hausdorff dimension equal $D(H) = \ln 2 / \ln 3 \approx 0.63$ and a topological dimension equal zero and in addition the length of this set is zero, i.e. it is a zero measure geometrical-topological structure. This is basically the zero set which models in E-infinity theory the pre-quantum particle. The second set is the white gaps between the black Cantorian points of the zero set [36]-[45]. The Hausdorff dimension of this set is simply one minus the Hausdorff dimension of the zero (black) Cantor set, i.e. $1 - 0.63 \approx 0.37$. The measure of this set on the other hand is equal to one minus zero which is then equal one indicating a fat Cantor set as shown in Fig. 2 [72]. Since this set represents the gaps in the "black" set, it is essentially an empty set with a negative topological empty set dimension equal minus one. It follows then that this "gaps" Cantor set is the empty set which models the pre-quantum wave in E-infinity theory [10], [13], [28]. The situation remains the same for a random Cantor set (see Fig. 3) except that $D(H)$ goes to $\phi^2 = (\sqrt{5} - 1)/2$ and thus $1 - D(H)$ goes to $\phi^2 = 0.381966$ instead of 0.37 [36]-[63].

It follows then from the geometry and topology of the above that each black point is essentially a pre-quantum particle with a positive topological attracting pressure equal to the Hausdorff dimension of the zero set which is ϕ while the white gap between them is the empty set modelling the pre-quantum wave with an opposing topological pressure equal to the Hausdorff of the empty set, namely ϕ^2 [63]-[76]. Consequently we have a resulting net pressure equal $\phi - \phi^2 = \phi^3$ which we can view as the topological local Casimir force. In turn ϕ^3 is equal to the Hausdorff dimension of the cobordism [35], [42], i.e. the surface of the quantum wave as well as being the expectation value of the multi-fractal surrounding of the quantum wave giving rise to the core of quantum spacetime as is obvious from the relation [63]-[76]

$$\begin{aligned} d_c^{(4)} &= (1/\phi)^{4-1} \\ &= 1/\phi^3 \\ &= (1/\phi)^3 \\ &= 4 + \phi^3 \end{aligned} \quad (5)$$

where $D(T) = 4$ is the topological dimension and $D(H) = 4 + \phi^3$ is the expectation Hausdorff dimension of quantum spacetime. We conclude that the above is an accurate one dimensional Cantorian spacetime which could now be expanded to the fully fledged 5 dimensional Kaluza-Klein fractal spacetime of our reality [63]-[76]. That way the pressure ϕ^2 behind the local Casimir pressure $\phi - \phi^2 = \phi^3$ becomes a five dimensional union ϕ^2 empty sets equal $(5)(\phi^2)$ which leads to a dark energy density equal to [63]-[76].

$$\begin{aligned} E(D) &= (5\phi^2) \left(\frac{1}{2} m \right) (v \rightarrow c)^2 \\ &= (5\phi^2/2) mc^2 \end{aligned} \quad (6)$$

This is self evidently the exact dark energy density found via accurate measurement and cosmological observation. Our second most import conclusion is that the Casimir energy is the local form of dark energy which by the well known theorem of Dvoretzky must be concentrated at the boundary of the holographic boundary of the universe [76]. This is one of the reasons why it cannot be detected locally in direct experiments although we know of its presence directly via the observed accelerated cosmic expansion [37]-[49].

IV. TO BE OR NOT TO BE A CANTOR SET IN MATHEMATICS AND PHYSICS

It is an illusion to think that a Cantor set is intuitively simple. A Cantor set is in fact both the simplest and the most mind boggling thing that there is on the foundational level of pure mathematics and mathematical physics.

To start with a Cantor set construction produces at the end two contradictory Cantor sets. Let us look at the classical iterative procedure of Figs. (1-3) which is rather familiar to those working in any of the many branches of

nonlinear dynamics, chaos and fractals [1]-[5], [64]-[76]. At the final stage after a theoretically infinite number of iterations we end up with an uncountable infinite number of black Cantorian “points” separated by an equally uncountable infinite number of white gaps. For the black set we have a truly contradictory existence because the length of this set is clearly zero, i.e. it is “measure” zero and consequently in the limit it is not there in any physical sense yet this set which is zero topological dimensions, has another substantial dimension, namely a Hausdorff dimension equal $\ln 2 / \ln 3 \approx 0.63$. For a randomly constructed Mauldin-Williams Cantor set [10] the situation is not different where we find that the Hausdorff dimension is equal $\phi = (\sqrt{5} - 1) / 2 = 0.618$ which is not

much smaller than the 0.63 of the classical Cantor triadic set [10]. In defence of the convoluted logic of this situation, one may argue that this is a Hausdorff dimension and more related to information and entropy while the familiar topological dimension of this Cantor set is following the Menger-Urysohn dimension theory is really zero. This argument however would not reduce the situation to a more intuitive level but would in fact increase the non-intuitive character of a Cantor set [1]-[10]. In this case we could still be persistent and point out that there is nothing called Cantor points because it is just a matter of scale and when a so called Cantorian point is magnified, we retrieve yet again an entire Cantor set. That would then make some sense because we can deny that there will ever be a real moment when a Cantor set disappears except in our imagined mental picture at infinity, i.e. at a tomorrow while we know that this “domani” will truly never come! But then here comes the next outrageous point related to the white set which compared to the black set is a fat Cantor set because it has a positive measure. Let us pause a moment and recall that we started our Cantor sets construction by a unit interval represented by a line of a topological and Hausdorff dimension coinciding and equal to unity. Similarly the length of this line is also unity. Now to construct the black set we must subtract the total length of this black set from the $\ell = 1$ and since the black set is measure zero [1]-[10], the white set would remain with a length equal unity. On the other hand for the Hausdorff dimension we have to subtract ϕ of the black set from unity and find the Hausdorff dimension of the fat white set to be $1 - \phi = \phi^2$. So far so good, but it remains to find the topological Menger-Urysohn dimension corresponding to ϕ^2 [40]-[45].

The safe way for finding out this quantity is to use the dimensional function of von Neumann and Connes and find to our mild surprise that it does represent again a challenge for classical intuition because the topological dimension corresponding to ϕ^2 Hausdorff dimension is neither zero nor one but rather minus one. Of course naïve expectation would have persuaded us to believe that the topological dimension of the dark set is zero so that we need to subtract zero from the topological dimension of the original “line” interval so that we have the dimension one. However such a conclusion would be mathematically

totally incorrect because the dimension one is reserved for a classical line while the white or fat Cantor set is not a classical line for the simple reason that it contains in it infinitely many black holes forming our thin black Cantor set. To make a long story short, the resolution of this contradiction is found in the topological concept of neighbourhood. It is a well established topological concept that the neighbourhood of a classical point with zero topological dimensions is an empty set with a minus one topological Menger-Urysohn deductive dimension [10] so that we may conclude that first the fat white Cantor set is an empty set and second this empty set is the cobordism, i.e. the surface of the thin black Cantor set [1]-[10]. The final magnificent outcome shows a totally pathological situation to be generic and at the root of not only mathematics but also physics. Quantum physics and cosmology are full with counter intuitive and classical paradoxical theoretically and experimentally well founded results such as quantum entanglement and dark energy. Nature, as seen within the limitation of mans' brain and logic, seems to be fundamentally pathological. Similar to treating bacterial infections using the same weapon, namely antibiotics, nothing is better suited to deal with the natural pathology of nature than the generic pathology of Cantor sets and multi-fractals which explains the undoubted success of E-infinity theory [10] for explaining what quantum entanglement and dark energy are and why $E = mc^2$ is actually the sum of two quantum components, namely $E(O) = mc^2/22$ for ordinary energy [36]-[45] and $E(D) = mc^2(21/22)$ for dark energy so that at the end we have the energy of a quantum particle $E(O)$ added to the energy of a quantum wave $E(D)$ giving us the maximal E of Einstein (see Fig. 4 a and b) [36]-[45]

$$E = E(O) + E(D) \\ = mc^2/22 + mc^2(21/22) = mc^2 \quad (7)$$

In this sense $E(O)$ could be regarded as the position energy and $E(D)$ as the kinetic energy while $E = mc^2$ is the maximal total energy with the possible interpretation of converting mass to energy, only in theory or also in actual real world, but this is not the point we want to discuss in the present context. To enhance understanding of the role of fractals in the present work and for a deeper understanding of the role of dimensions, the reader is referred to Appendix 1-3. We should also stress at this point the fundamental role played by Hardy's quantum entanglement [74] in dark energy [54], [61].

V. THE LOGARITHMIC SPIRAL CONNECTING QUANTUM FIELD THEORY WITH E-INFINITY THEORY

The logarithmic spiral with its self similar geometrical structure creating golden mean rectangles step by step as shown in Fig. 5 is a remarkable visual confirmation that E-infinity space, quantum field theory logarithmic scaling and the golden mean scaling of E-infinity theory [6] are most probably deeply connected as careful mathematical analysis would reveal [6]-[10]. This is thus more than an intuitive hunch and is clearly the *raison d'être* of the

success of standard renormalization equations such as [26]-[34], [86], [90]-[93]

$$\bar{\alpha}_u = \bar{\alpha}_1 + \bar{\alpha}_2 + \rho \ell m \frac{Mu}{Mx} \quad (8)$$

and its exact E-infinity counterpart [90]-[93]

$$\bar{\alpha}_u = \bar{\alpha}_1 + \bar{\alpha}_2 + (\bar{\alpha}_o)(\phi)^n \quad (9)$$

This subject is discussed in considerable details in refs. [26]-[34]. Here we give only the analysis for a grand unification for non-super symmetric $\rho=1$ and the super symmetric $\rho=1/2$ case. The unification energy in this case corresponds to a 'tHooft-Polyakov monopole $M_u \cong 10^{16}$ GeV while the reference mass is taken as usual to be a typical electroweak scale, namely $m_z = 91$ GeV. That way we find [88][90]-[93].

$$\bar{\alpha}_g = (9+1) + (1) \ell n \frac{10^{16} \text{ GeV}}{91 \text{ GeV}} \\ = 10 + 32.33 \\ = 42.3 \quad (10)$$

which is very close to the expected accurate value [88]-[91]. For the super symmetric case we find [93]

$$\bar{\alpha}_{gs} = (9+1) + \left(\frac{1}{2}\right)(32.33) \\ = 10 + 16.165 \\ = 26.165 \quad (11)$$

Again it is an excellent result compared to the exact one, namely $26 + k = 26.180339$ so far for the logarithmic scaling of quantum field theory. For the E-infinity golden mean scaling on the other hand we find [86], [91]-[93].

$$\bar{\alpha}_{gs} = (9+1) + (\bar{\alpha}_o/2)(\phi) \\ = (10) + [(137 + k_o)/2](\phi) \\ = 42.36.. \quad (12) \\ \text{and [91]-[93]} \\ \bar{\alpha}_{gs} = (10) + (\bar{\alpha}_o/2)(\phi)2 \\ = 26 + k \\ = 26.18033989 \quad (13)$$

exactly as expected [26-34]. Note that this is the same result as that which we obtain for quantum gravity [26]-[34], [86]. In other words when we include super symmetry we automatically include gravity in our calculation. This becomes clear when we take in the above calculation $M_u \cong (10)^{19}$ instead of $(10)^{16}$ and replace $m_z = 91$ GeV by that of the electron $m_e = 0.511$ GeV. That way we find that [26]-[34], [86], [91]-[93].

$$\bar{\alpha}_3 = 0, \bar{\alpha}_4 = 1 \text{ and}$$

$$\bar{\alpha}_u = 1 + \frac{1}{2} \ell n \frac{10^{19} \text{ GeV}}{[(0.511)/1000] \text{ GeV}} \\ 1 + \frac{1}{2} \ell n (1.9564)(10)^{22} \\ 1 + \frac{1}{2} (22 \ell n 10) + \ell n 1.9569 \\ = 1 + (51.32)/2 \\ = 26.66 \quad (14)$$

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$$1 + \frac{1}{2} \ell n(1.9564)(10)^{22}$$

$$1 + \frac{1}{2} (22 \ell n 10) + \ell n 1.9569$$

$$= 1 + (51.32)/2$$

$$= 26.66$$

using the golden mean scaling. On the other hand things are quite straight forward leading to [88], [91]-[93].

$$\bar{\alpha}_u = (\bar{\alpha}_o/2)(\phi)^2 \quad (15)$$

$$= 26 + k$$

just as in the grand unification case [91]-[93].

VI. THE RELATION BETWEEN VOLUME AND ENERGY

Clearly there can be no energy without spacetime volume to contain it. This statement remains true even if volume and energy are one and the same thing as in general relativity curvature of spacetime and gravity are one and the same. Now we have two volumes. First we have the volume of the five dimensional pre-quantum particle [26]-[34]. This is a conventional multiplicative volume equal $(\phi)^5$ where the 5 is $D = 5$ of Kaluza-Klein spacetime [46] and ϕ is the Hausdorff dimension of the pre-quantum particle interpreted as a measure [26]-[34]. Second we have the volume of the pre-quantum wave. This is an additive volume being more of the higher dimensional circumferential length of the wave enveloping the particle in $D = 5$ which means $5\phi^2$ where ϕ^2 is taken as a measure for the pre-quantum wave. Now since the wave is the surface of the particles, we have as many particles as waves and the particle wave duality leads therefore to the obvious average volume of a quantum particle, namely the arithmetic mean of the multiplicative particle volume and the additive wave volume from which we find [35], [42], [60], [62].

$$\begin{aligned} \langle Vol \rangle &= \frac{1}{2}(\phi^5 + 5\phi^2) \\ &= \frac{1}{2}(2) \\ &= 1 \end{aligned} \quad (16)$$

Consequently $E = mc^2$ could be rewritten as [62], [76].

$$\begin{aligned} E &= (5\phi^2)mc^2 + \frac{5\phi^2}{2}mc^2 \\ &= E(O) + E(D) \\ &= mc^2 \end{aligned} \quad (17)$$

as claimed by us at the very beginning of this section [92]. In the next section we aim to show that the principle of self similarity also holds true for Newton gravity and quantum gravity [61]-[63].

VII. DARK ENERGY AND ORDINARY ENERGY DENSITY OF THE COSMOS FROM DVORETZKY'S THEOREM OF BANACH SPACES

There are a few things that we need to know before noticing at once that the Dvoretzky theorem [34], [39] is already the answer to our main question regarding dark energy question. First we must know that Einstein's energy $E = mc^2$ is the maximal energy density possible in the universe when m is the mass and c is the speed of light [34], [39]. This formula is simply a similarity gauging of Newton's $E = \frac{1}{2}mv^2$ when $\frac{1}{2} \rightarrow 1$ and $v \rightarrow c$. Second we need to reason that a spherical multi-dimensional ball is the most likely shape of our universe. Thus our universe must be at least a five dimensional sphere which may be regarded as a sufficiently high dimensionality to make Dvoretzky's theorem work. Now we see that the theorem employs a volume measure concentration at the surface of the sphere amounting to 96% while the rest of the universe contains only 4% of the volume. Since energy must be proportional to the volume of spacetime, it follows directly that E is 96% at the end of the universe, i.e. at the boundary of the holographic boundary [34]-[39], [43]-[45].

It does not seem far fetched at all that our real spacetime must be at least five dimensional. The 4 dimensions of Einstein are self evidently 3 space and one time dimension which were merely fused together in special and general relativity. On the other hand the phenomenal success of Kaluza-Klein's five dimensional theories, let alone 10 dimensional super strings and eleven dimensional super gravity and Witten's M-theory [47] are all pointing in this direction and seems to reinforce the belief that Dvoretzky's theorem applies indeed to our universe. The measurement of ordinary energy $E(O) = 4.5\%$ and the conjectured dark energy presumed to be the cause for accelerated cosmic expansion consistent with $E(D) = 95.5\%$ with the prediction of Dvoretzky's theorem, namely $E(O) = 4\%$ inside the universe which means local ordinary energy density and dark energy density $E(D) = 96\%$ at the boundary of the universe causing the expansion must be regarded as an excellent experimental confirmation of Dvoretzky's theorem applicability to our cosmic fractal "manifold" [42]. We could give a rather short and neat theoretical analysis confirm the above without appealing in any direct or indirect way to Dvoretzky's theory. To do this we start from E-infinity theory [34], [39] at the observable electroweak energy scale for which we have [88], [91]-[93].

$$\bar{\alpha}_o = (1/\phi)\bar{\alpha}_1 + \bar{\alpha}_2 + \bar{\alpha}_3 + \bar{\alpha}_4 \quad (18)$$

where $\phi = (\sqrt{5} - 1)/2$, $\bar{\alpha}_3 = 9$ and $\bar{\alpha}_4 = 1$. We may recall that $\bar{\alpha}_1$ is the inverse electromagnetic coupling, $\bar{\alpha}_2$ is the inverse weak force coupling, $\bar{\alpha}_3$ is the inverse strong coupling and $\bar{\alpha}_4$ is the maximal quantum gravity coupling all measured at the electroweak scale and results in the exact theoretical $\bar{\alpha}_0 = 137.082039325$ inverse electromagnetic fine structure "constant". It is important to

observe that $\bar{\alpha}_i$ seen as dimensions of a corresponding symmetry groups manifold add to exactly 100 because

$$\sum_{i=1}^4 \bar{\alpha}_i = 60 + 30 + 9 + 1 = 100 \quad (19)$$

This could be interpreted as the normed total dimensions of the universe and could be sub-divided into the following [40]-[43]:

1. $D_1 = 4\%$ for spacetime of ordinary energy
2. $D_2 = 26 - 4 = 22\%$ for compactified bosonic dimensions of dark matter energy
3. $D_3 = 100 - 26 = 74\%$ for the pure dark energy.

Again this agrees with a very high accuracy with the COBE, WMAP and Type 1a supernova measurements and observations and of course, also with Dvoretzky's theorem [39]-[41].

VIII. THE SELF SIMILARITY BEHIND

$$\begin{aligned} \text{NEWTON'S } E &= \frac{1}{2} MV^2, \text{ EINSTEIN'S } E = \\ \text{MC}^2 \text{ AND EL NASCHIE'S } E(O) &= \text{MC}^2/22, E(D) \\ &= \text{MC}^2(21/22) \text{ AND } E(O) + E(D) = \text{MC}^2 \end{aligned}$$

A cow is definitely not a sphere but for all topological purposes it may be approximated to a sphere and glossing over biologically indispensable holes as well as the four stretched legs. This may be a drastic way to start arguing for an overwhelming self similarity of the cosmos and may also be the mathematical equations describing the cosmos. From this somewhat too general view point to be of any practical value, the manifest similarity between Newton's kinetic energy $E = \frac{1}{2} mv^2$ on the one side and celebrated Einstein's maximal energy equation $E = mc^2$ and El Naschie's two components equations $E(O) = mc^2/22$ and $E(D) = mc^2(21/22)$ which sum up to that of Einstein $E(O) + E(D) = mc^2$ [36]-[43], on the other side it is difficult not to dream of finding a universal scaling law which explains that all celestial objects as small as a meteorite and as large as a galaxy may be regarded topologically as spheres with three spatial dimensions and two dimensional surface [36]-[43].

The first step in this direction could have been showing that four dimensions seems to be the expectation number of weighted integers from zero to infinity in the sense of E-infinity theory. This is really the reason behind the E-infinity Hausdorff dimension expectation [36]-[43] value

$$\begin{aligned} \langle d_i \rangle &= (1/\phi)^3 \\ &= 4 + \phi^3 \end{aligned} \quad (20)$$

where $4 + \phi^3$ is also the Hausdorff dimension of so that we may write [10].

$$\begin{aligned} d_c^{(4)} &= \langle d_c \rangle = 4 + \phi^3 \\ &= 4.23606 \\ &\simeq 4 \end{aligned} \quad (21)$$

In such a space with infinite hierarchal topological dimensions, self similarity is basic and most fundamental so that it should not come as a surprise to observe that all

formulas related to something as fundamental as energy (E) should be subject to a minimum of self similarity or self affinity. To stress the point we recall that E-infinity spacetime is made of an infinite number of unions at an intersection of random Cantor sets with a Hausdorff dimension equal to ϕ^n where n runs from zero to infinity.

In turn Hausdorff dimensions are a measure of complexity and therefore are deeply related to entropy and consequently energy ergo the property of Cantorian self similarity is expected to be passed over to energy which means passed over to the equations defining energy. Could such a thought have passed in the mind of the great Albert Einstein? This might be idle or even far fetched unscientific speculation. However there is a curious and partially amusing story which could lead one to think that such ideas may really have crossed the great mind of Einstein (see Fig. 4 a and b). The said story is recorded in the classical and wholly enchanting book of the notable German American physicist Manfred Schroeder "Fractals, Chaos and Power Laws". In there Schroeder tells the story of how Einstein found a self similarity proof for the theorem of Pythagoras and shows that this part of Einstein's proof includes a formally identical formula to $E = mc^2$ (see Fig. 6). However in all fairness to Schroeder he said that it is a mere coincidence and has nothing to do with the real $E = mc^2$ except the appearance and that $E = mc^2$ can be meaningful only in the context of Lorentzian invariance. On this count however we beg to differ. Admittedly biased by our own result $E = mc^2/22 + mc^2(21/22)$ [36]-[43] and our research on the subject we see here far more deep reasons than a coincidence as we will momentarily explain [37].

Let us start by quoting what is written on page 4 of the excellent book of Schroeder [6]

"The 'resemblance' of equation 3 to Einstein's later discovery, his famous $E = mc^2$, is of course entirely fortuitous. The equivalence of mass m and energy E which is at the basis of nuclear power in all its guises is a consequence of Lorentz invariance. This invariance which underlies special relativity was predicted by Einstein in 1905 after it seems, several false starts....."

Now analysing the above in a liberal way consistent with the big fuzzy picture approach of E-infinity, we can take the resemblance to mean self similarity or self affinity while the Lorentz invariance could be understood as a gauge which means scaling and renormalization and these notions are in turn just another more or less sophisticated form of self similarity which Einstein used to devise an alternative proof for the Pythagoras theorem [6]. In fact this theorem of Pythagoras is indispensable in giving a simple derivation of $E = mc^2$ as used in another excellent popular book on the subject by Brian Cox and Jeff Forshaw "Why Does $E = mc^2$ " [36]-[43]. With all of that in the back of our minds, looking at $E = \frac{1}{2} mv^2$ of Newton compared to $E = mc^2$ of Einstein we do not need much persuasion to see that it is only a matter of scale where the $\frac{1}{2} \rightarrow 1$ because we are integrating in the case of $v = c$ over a constant velocity, namely that of light and $v \rightarrow c$ is also a matter of scale. For this reason maximal energy equation $E = mc^2$ and El Naschie's two components equations $E(O) =$

$(1/22) mc^2$ and $E(D) = (21/22) mc^2$ could be seen as scaling of $E = mc^2$ when due to certain conditions the Lorentzian factor $\gamma = 1$ of maximal "Einstein" energy takes two other values $\gamma(O) = 1/22$ and $\gamma(D) = 21/22$ for ordinary and dark energy respectively. Seen in this way it is quite conceivable, even though it is not very likely, that Einstein could have discovered the general form of E , namely [36]-[43] $E = (mc^2/22) + mc^2(21/22) = mc^2$ long ago. After all it is sometimes good to think superficially. This dialectic statement could be illustrated by a well known observation of Einstein himself who explained time as that quantity that we measure with a clock. A second not so good example of the above is that when v is constant then it goes out of the integration belonging to the definition of energy and $1/2$ becomes unity and v may be replaced by a new notation, namely the constant c so that at the end $E = 1/2 mv^2$ is replaced by $E = mc^2$. From a formal view point that is really all and our conclusion is that self similarity and generalized equivalence relations [106], [108]-[111] are far more powerful scientific tools than we could have imagined before the advent of chaos, fractals and nonlinear dynamics.

To end this section we should add that according to Manfred Schroeder, the story of Einstein's proof of Pythagoras' theorem was recounted to him by SchneiovLifson of the Weizmann Institute in Tel Aviv who had it from Einstein's assistant Ernst Strauss who was told the story by Albert Einstein himself [6]. Similar to Newton discovering gravity by watching an apple falling from a tree, the Einstein story is so nice that it should be true even if it were not.

IX. SELF SIMILARITY IN NEWTON GRAVITY AND QUANTUM GRAVITY

It is remarkable that our self similarity principle which we used to justify moving from $E = 1/2 mv^2$ to $E = (mc^2/22) + mc^2(21/22) = mc^2$ can be extended to the realm of Newton gravity and quantum gravity [36]-[43]. In the particular case considered here, quantum gravity inverse coupling constant can easily be seen following our E-infinity derivation to be the ratio between the total number of isometries of the E8E8 quantum spacetime and the total number of isometries at the energy scale of the standard model. This means [36]-[43].

$$\begin{aligned} \bar{\alpha}_{Qg} &= \frac{|E8E8|}{|SU(3)SU(2)U(1)|} \\ &\approx \frac{496}{12} \\ &\approx 41.3 \end{aligned} \quad (22)$$

which is very close to most of the results found in the literature in the non-super symmetric case. Similarly the number of isometries of the manifold combining classical gravity with all other fundamental forces is a staggering number, namely 10^{19} as we will show later on. We also know that at the point of total unification of all fundamental forces we are simply dealing with the Planck energy scale and that the coupling constant in this case is a

maximum equal 1. This is the only degree of freedom. Thus the equation corresponding to [47].

$$\bar{\alpha}_{Qg} = \frac{496}{12} \quad (23)$$

must now be [74].

$$\bar{\alpha}_{Planck} \approx \frac{10^{19}}{\text{Newton degrees of freedom}} \quad (24)$$

That means

$$1 \approx \frac{10^{19}}{\text{Newton degrees of freedom}} \quad (25)$$

In other words Newton's degrees of freedom is $\approx 10^{19}$. The next step is somewhat subtle and needs delicate consideration. We know that $SU(3) SU(2) U(1) = 12$ is actually the square root of the inverse electromagnetic constant. The accurate transfinite expression is really [26]-[36].

$$\begin{aligned} |SU(3)SU(2)U(1)| &= \sqrt{\bar{\alpha}_o} \\ &= \sqrt{137.0820393} \\ &= 11.7082039325 \\ &\approx 12 \end{aligned} \quad (26)$$

It follows then that

$$\sqrt{\bar{\alpha}_{(Newton)}} \approx 10^{19} \quad (27)$$

That means

$$\begin{aligned} \bar{\alpha}_{(Newton)} &\approx (10^{19})^2 \\ &\approx 10^{38} \end{aligned} \quad (28)$$

in a rather excellent agreement with the inverse value of the dimensionless Newtonian gravity. As for the spacetime manifold with 10^{19} isometries which we mentioned earlier on, this was investigated about eight years ago [26]-[36] and named Finkelstein-Clifford manifold. The main idea behind this very large number starts from looking at how many Planck length mini black holes 10^{-33} cm could fit into a proton wave length 10^{-14} cm which means $10^{-33}/10^{-14} = 10^{19}$ or alternatively how many proton masses $m \approx 939$ Mev fits into a Planck mass 10^{19} Gev which means nearly 10^{19} Gev/1Gev = 10^{19} [26]-[36].

X. CONCLUSION

Starting from nonlinear dynamics, chaos and fractals and linking that with the pure mathematical results of the theory of equivalence relations [108]-[111], the present work takes very seriously several ideas revolving around self similarity and scaling. Within such theories we regard nature as generically fuzzy and almost pathological at the fundamental level [65], [107]. We take then the view that these ideas are by no means entirely new but were thought about deeply probably by Albert Einstein and for sure by the great Russian-American academician G.I. Barenblatt who wrote in his book entitled "Simply Scaling" [106] "every mathematical model is based on 'intermediate asymptotic'". Adding this to what the French Fields Medalist and exceptional mathematician and theoretical physicist A. Connes [107] wrote about the pathological Penrose universe [46], [65] the main thrust of the present

paper becomes clearer. From there to arrive from Newton's kinetic energy to the new Einstein ordinary energy and dark energy density of the universe it is exceedingly shorter than the long way to Tipperary.

APPENDIX 1

Illustration of the first five iterations of the construction of a deterministic Cantor set

Referring to Fig. 1 and with the aim of having a clearer representation as well as to introduce the link between measure and set, the segments in the figure were chosen as bars of fixed width and consequently representing a uniform density distributed over the support set. As the number of iterations goes to infinity, the Hausdorff dimension goes to $\ln 2 / \ln 3 = 0.6309297536 \approx 0.63$.

In other words nothing remains of the initial black line of unit length except an uncountable infinite number of "Cantorian" black points (or bars) with a total length equal zero. Thus we could imagine the black line to have been replaced by a white line of the length one minus zero which is the original line of the length of the black line. Therefore we could say that we have two "Cantor" sets. The first is an infinite number of black "points" with length, i.e. measure equal zero and a Hausdorff dimension equal 0.63 apart of a topological dimension equal zero because it consists only of points at the limiting infinity.

The second set by contrast is a white line of a unit length of a Hausdorff dimension equal $1 - 0.63 \approx 0.37$. It is important now to find a consistent topological dimension for this set which represents the totality of the gaps which we created in the initial black line by removing the middle third iteratively in the familiar fractal construction manner. Since the white "set" must be either nothing or an empty set, then its dimension must be negative. Formally it is minus one as is clear from von Neumann-Connes formula or it is minus infinity for the same reasons but taking into account the special situation of a one dimensional universe rather than the real infinite dimensional spacetime of Cantorian E-infinity theory [7]-[17].

APPENDIX 2

Generalizing the Classical Cantor Set to Two Dimensions

The two dimensional counterpart of the one dimensional triadic Cantor set is neither the two dimensional Swiss flag fractal (see Fig. 7) with $D_H = (2)(\ln 2 / \ln 3)$ nor is it the Sierpinski carpet (see Fig. 8) with $D_H = \ln 8 / \ln 3 = 3(\ln 2 / \ln 3)$ but rather the Sierpinski gasket (see Fig. 9) with D_H equal the inverse of the Hausdorff dimension of the Cantor set, namely $\ln 3 / \ln 2 = 1.584962501$. Again the white triangles correspond to empty sets with negative topological dimensions. The generic E-infinity fractal corresponding to this gasket is the two dimensional random Cantor set which is also found from the inverse of ϕ to give us

$1/\phi = 1 + \phi = 1.618033989$ which is rather close to $\ln 3 / \ln 2 = 1.584962501$ (see Refs. [1]-[6]).

APPENDIX 3

Generalizing the classical Cantor set to three dimensions

Referring to Fig. 10 and Fig. 11 we can say the following:

The generic generalization of Cantor's triadic set to 3 dimensions is neither that shown in Fig. 10 nor is the Menger sponge of Fig. 11 which is a generalization of the Sierpinski carpet shown in Fig. 8. On the other hand a generalization of a random Cantor set of the Mauldin-Williams type, i.e. is easily achieved for any dimension using von Neumann-Connes dimensional function or equivalently the bijection formula of E-infinity theory $d_c^{(n)} = (1/\phi)^{n-1}$ where n is the topological dimension. That way a "generic" random Menger sponge dimension should be $D(H) = (1/\phi)^{3-1} = 2 + \phi = 2.618033989$ and in four dimensions it is $D(H) = (1/\phi)^4 = 4 + \phi^3$ (for details see Refs. [1]-[6] and Ref. [10]).

APPENDIX 4

Equivalence Relations in Mathematics and Physics

The interest of the present work in the pure mathematics and mathematical logic of equivalence relations stem from numerous expected and less expected connections to theoretical and mathematical physics [1]-[111]. For instance we know that equivalence relations is a binary relation that is reflexive as well as symmetric and transfinite, all simultaneously [108]. From that we can deduce that this equivalence relation will result in a partition of a set of equivalence classes. In other words, it results in a quotient set of a quotient space. A generic quotient space on the other hand is the x space representing Penrose fractal tiling universe [7]-[17]. In turn this space is a prototype of A. Connes' noncommutative geometry as well as a realization of E-infinity Cantorian spacetime with the same dimensional function [75]. In other words, it is akin to a similarity classifier with a golden mean [109] and could be drawn into understanding the similarity of Einstein's formula $E = mc^2$, as well as the Planck radiation expression $E = hf$ and possibly Newton's second law $F = ma$ as well as the similarity of Newton's kinetic energy $E = \frac{1}{2} mv^2$ and our dissected Einstein formula $E = (mc^2/22) + mc^2(21/22) = mc^2$ [53]-[59].

X. ACKNOWLEDGMENT

The totally empty set notwithstanding, we are not living in a scientific vacuum. There are hardly any 'new' ideas which were not anticipated in one form or another by many thinkers, philosophers, scientists or engineers. We frequently omit mentioning this particularly when we

experience the overwhelming feeling of discovering something new. Forgetting to acknowledge those from whom we learned so much consciously and far more unconsciously is painful in both directions and the Author takes this occasion to deeply acknowledge the intellectual indebtedness to many of his teachers, colleagues, associates as well as past and present students from whom he learned a great deal. It is of course too late to say it more than for history regarding those who are unforgettable who are sadly no longer with us, particularly Prof. Alf Pflüger and Prof. Theodor Lehmann. The influence and guidance I received from Prof. C.F. Weizecker, Prof. W. Heisenberg and Prof. Werner Martienssen cannot be overstated. Luckily there are many who I can still reach directly with gratitude for their profound work which guided me as it guided many younger and older generations. In particular I must mention G. 'tHooft, D. Gross, L. Susskind and O. Rössler. The list of names is of course too long and space, as life, is too limited to mention them all.

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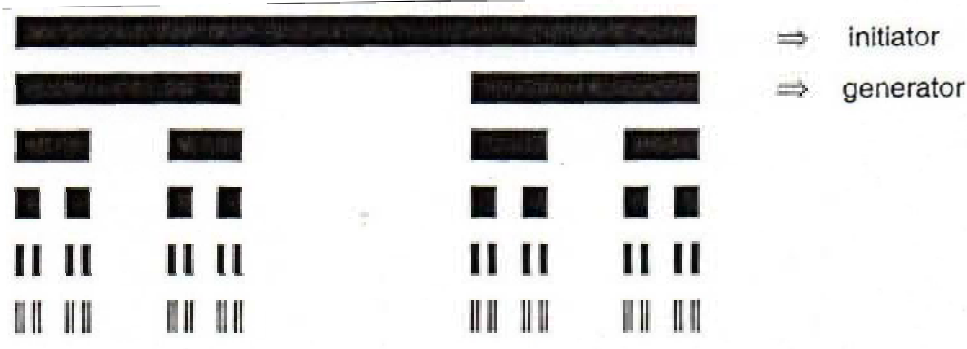


Fig. 1. A one dimensional Cantor universe (see Refs. [1, 2 and 6])

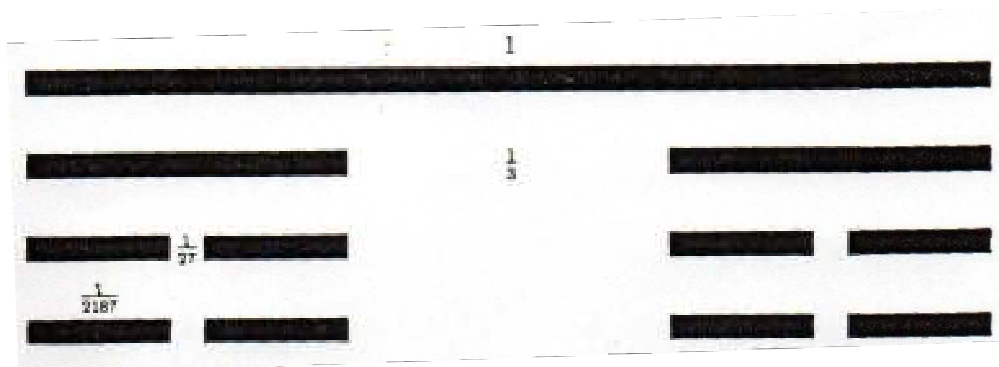


Fig. 2. Construction of a fat fractal. It has infinitely many holes but the remainders keep a total length that is greater than zero (see Ref.[6]).

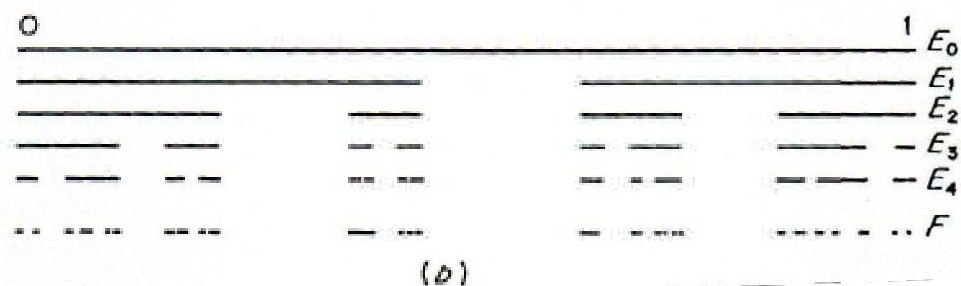
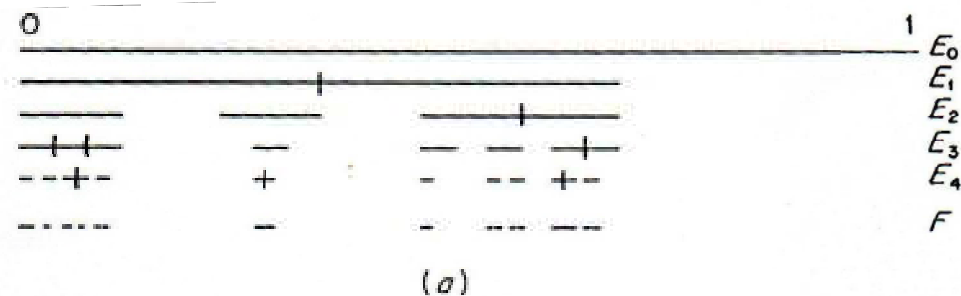


Fig. 3 a and b – Two random versions of the Cantor set. In (a) each interval is divided into three equal parts from which some are selected at random. In (b) each interval is replaced by two subintervals of random lengths.

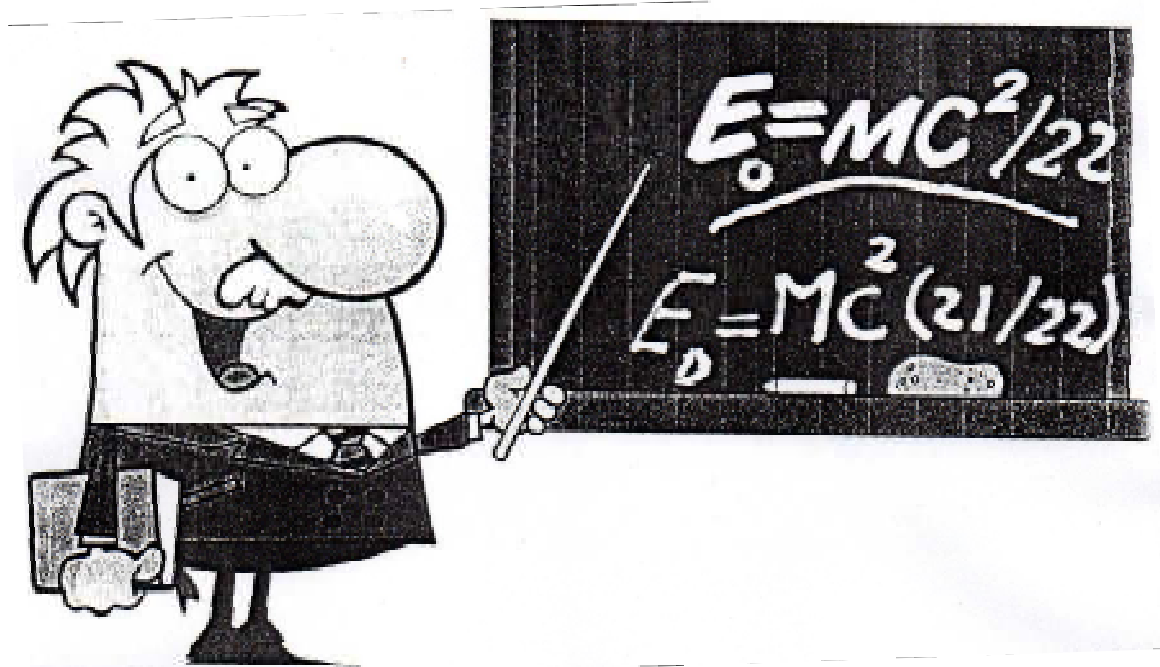


Fig. 4. a – A light hearted historical picture depicting the moment when the truly great Albert Einstein was about to discover E-infinity dark energy formula $E(D) = mc^2(21/22)$ as well as the ordinary energy formula of the same theory $E(O) = mc^2/22$. It seems possible that after giving an alternative proof for the theorem of Pythagoras based on the principle of self similarity that he realized that this theory and Newton's kinetic energy $E = \frac{1}{2}mv^2$ are self similar. That way he could have discovered the chaotic nonlinear dynamics beneath quantum mechanics and found El Naschie's dissection of $E = mc^2$ into its quantum components $E = (mc^2/22) + mc^2(21/22)$ (see Refs. [36 and 50]).

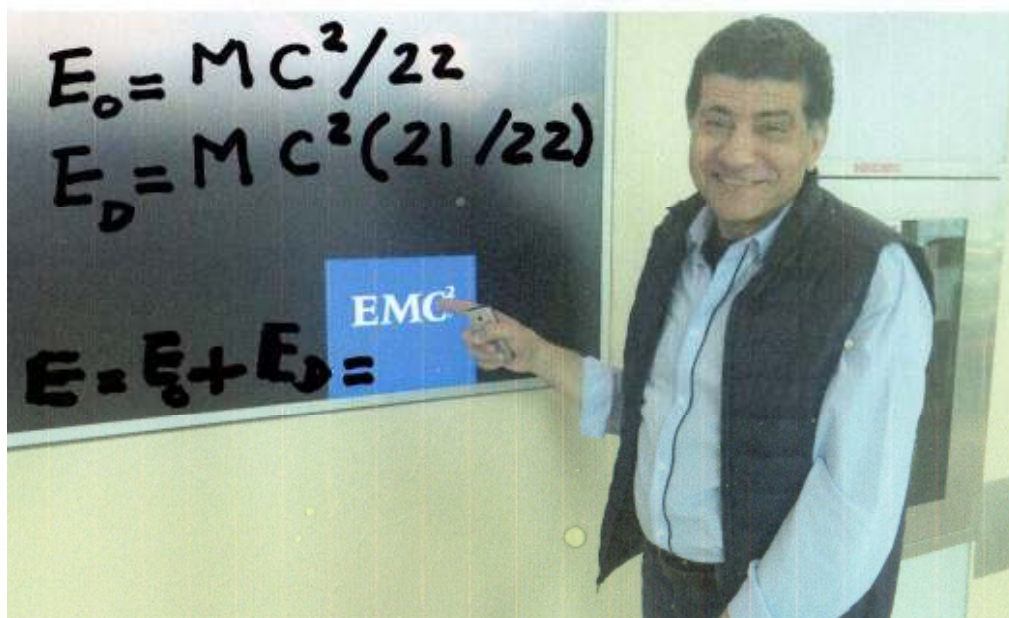


Fig. 4. b – Einstein's $E = mc^2$ is probably the most famous equation in science. In fact this photo was taken when the Author was passing in the corridors of Dubai airport in 2015 and was struck by seeing $E = mc^2$ on the walls. The rest, namely $E_0 = mc^2/22$ and $E_D = mc^2(21/22)$ as well as $E = E_0 + E_D = mc^2$ were superimposed later by the Author (see Refs. [50]and[62] as well as Appendix 4)

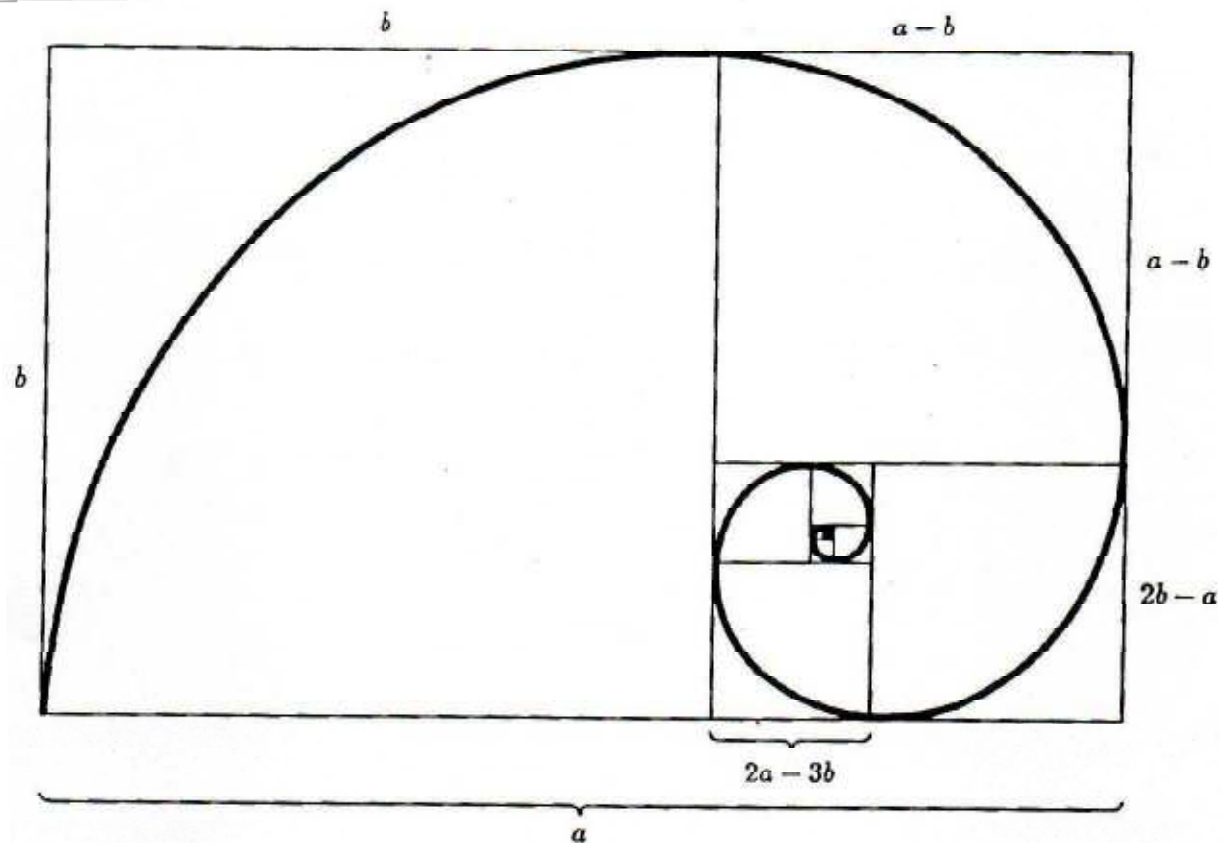


Fig. 5. A self similar succession of golden mean rectangles, produced by the logarithmic spiral. This is a clear hint at the deep connection between E-infinity golden mean scaling and the logarithmic scaling of quantum field theory (see Section 5 of the present paper as well as Appendix 4).

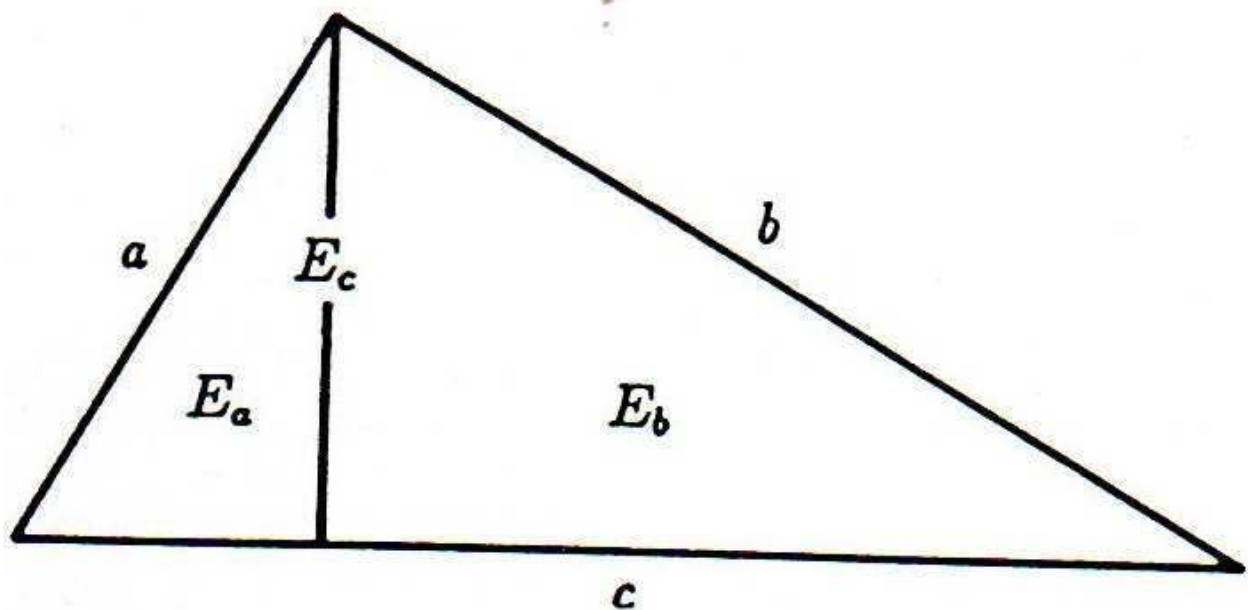


Fig. 6. Pythagoras' theorem sketch for proof by the 11 year old Einstein based on self similarity (see Section 8 of the present work as well as Ref. [6])

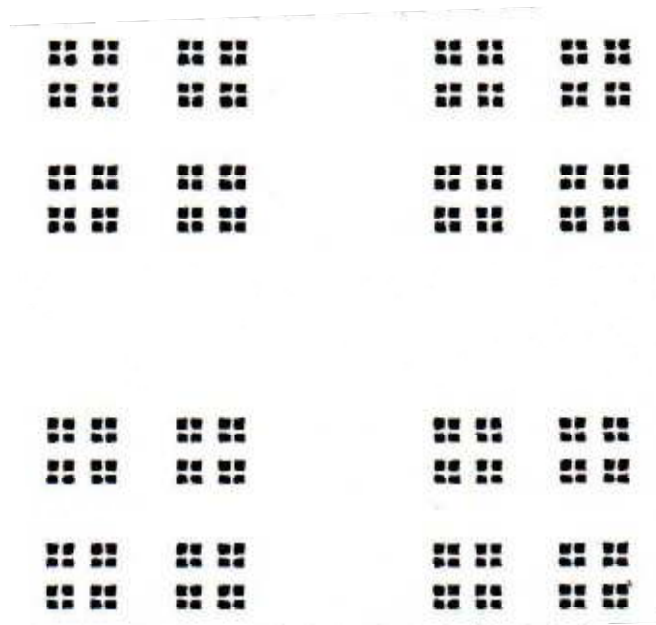


Fig. 7. The Swiss flag fractal which is the Cartesian product of two Cantor sets

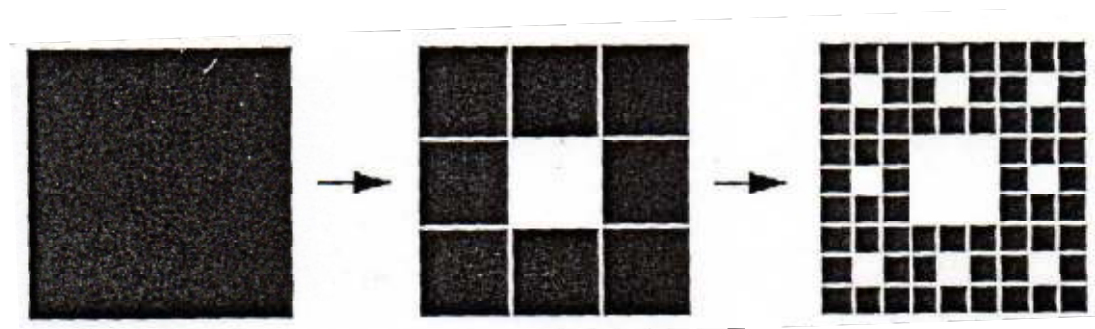


Fig. 8. The Sierpinski carpet is the two dimensional version of the Menger sponge. The generic case is found for a random fractal to that found from the bijection formula of E-infinity theory $d_c^{(n)} = (1/\phi)^{n-1}$ by setting $n = 2$ and finding that $d_c^{(2)} = (1/\phi) = 1.618033989$.

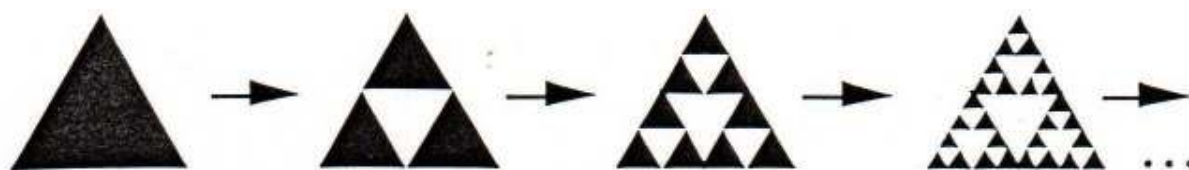


Fig. 9. The Sierpinski gasket as the right way to lift the one dimensional triadic Cantor set $D_{(H)} = \ln 2 / \ln 3$ to two dimensions. The Hausdorff dimension in this case is the inverse of $\ln 2 / \ln 3$, namely $\ln 3 / \ln 2$. Thus it obeys the dimensional function of von Neumann-Connes (see Ref. [107]).

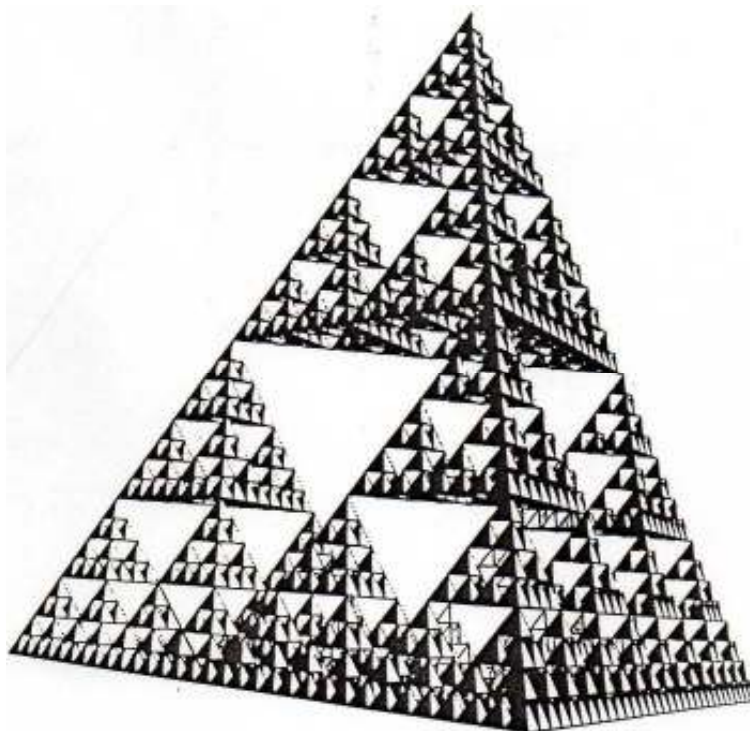
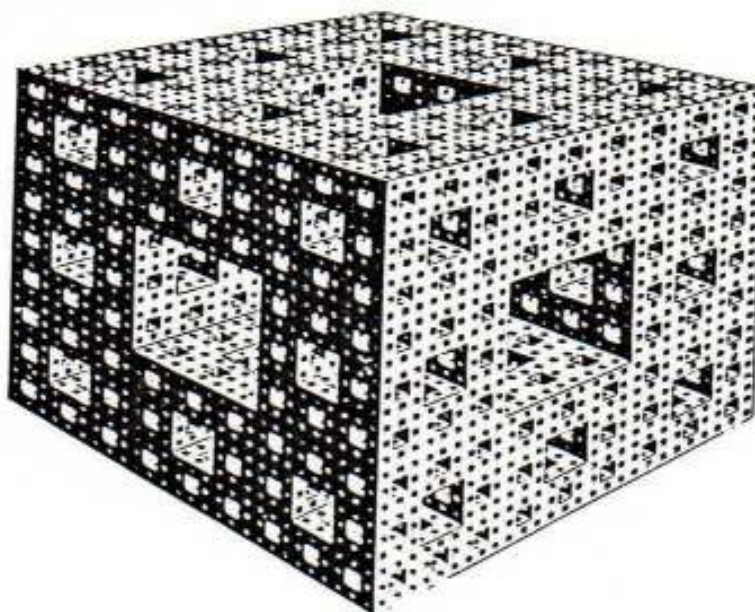


Fig. 10. A three dimensional version of the Sierpinski gasket. Its fractal dimension $\log 4 / \log 2 = 2$ has an integer value (2), albeit smaller than the dimension of the supporting space (3) (see Refs. [1]-[5]).



$$\text{Menger sponge } D = \log 20 / \log 3 \cong 2.73$$

Fig. 11 - The Menger sponge although it is not the real generalization of a Cantor set to three dimensions has a Hausdorff dimension quite close to the random three dimensional Cantor set $\left(D = (1/\phi)^2 \approx 2.618033989 \right)$. It is used in many applications including modelling three dimensional fractal space.