International Journal of Innovation in Science and Mathematics
Volume 5, Issue 3, ISSN (Online): 2347–9051

A Weaker Form of a Closed Map in Nano Topological Space

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Date of publication (dd/mm/yyyy): 02/06/2017

Abstract — **The aim of this paper is to introduce a new class of function in Nano Topological space called Nano weakly generalized closed map. The condition for a map to be Nano weakly generalized closed map is discussed.Nano weakly generalized open map is also introduced and studied a few basic properties. Also their relationship with already existing closed map and open map in Nano Topological space are investigated. Finally Nwg-homemorphism and Nwg* - homeomorphism are established and their properties are analyzed with suitable examples.**

Keywords — **Nano Topological Space, Nano Closed Sets, Nano Continuous Functions and Nano Closed Function.**

I. Introduction

In 1963 Levine[6]. had given an opening for the generalization of closed sets and he himself introduced the generalized closed set in this year. Njastad[11]. introduced a new class of open sets called α - open sets in topological spaces. Biswass[3]. introduced and investigated semi open sets in topological space.In 1978 Long and Herrington [7]. studied the properties of regular closed sets. Mashhour, Abd El-Monsef and El-Deep [8].investigated the characteristics of pre-open sets. Sundaram and Nagaveni [13]-[14] analyzed a weaker form of generalized closed set called wg-closed set in topology and extended their work to weakly generalized continuous and irresolute maps. In 2009 Parimelazhagan and Nagaveni [12]. investigated a weaker form of closed map namely weakly generalized closed map in minimal structure.

In 2013 Lellis Thivagar [4]. introduced a new class of topology namely Nano Topological Space using upper, lower approximations and boundary region of a subset of an universe using equivalence relation on it. And also discussed some of the nano forms of weakly open sets. The author[5]. has extended his work to Nano continuity and Homeomorphisms.

Bhuvaneswari and Mythili Gnanapriya[1]-[2]. studied the concept of generalization in Nano Topological space. Nagaveni and Bhuvaneswari [9]-[10]. used the weakly generalization in Nano topological space in 2014.

In this paper a new class of closed map in Nano Topological space called Nwg-closed map is introduced and some of its properties are analyzed.

Throughout this paper $(U,\tau_R(X))$ is a Nano Topological space with respect to X Where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R. $(V,\tau_{R'}(Y))$ is a Nano Topological space with respect to Y Where $Y \subseteq V$, R' is an equivalence relation on V, V/R' denotes the family of equivalence classes of V by R'. $(W,\tau_{R''}(Z))$ is a Nano Topological space with respect to Z Where $Z \subseteq W$ R'' is an equivalence relation on W, W/R'' denotes the family of equivalence classes of W by R''.

II. Preliminaries

This section is to recall some of the basic definitions and results that are useful for the study.

Definition:2.1[4] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$,

1. The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is defined by $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$. Where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is defined by $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and is defined by $B_R(X) = U_R(X) - L_R(X)$.

Proposition: 2.2 [4]

If (U, R) is an approximation space and $X, Y \subseteq U$ Then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\phi) = U_R(\phi) = \phi$ $L_R(U) = U_R(U) = U$
(iii) $U_R(X \cup Y) = L_R(X) \cup L_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$, $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R(X^c) = [L_R(X)]^c$
(ix) $L_R(X^c) = [U_R(X)]^c$
(x) $U_R U_R(X) = L_R U_R(X) = U_R(X)$
(xi) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition: 2.3[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$.

Then $\tau_R(X)$ satisfies the following axioms.

1. U *and* $\phi \in \tau_R(X)$.

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ forms a topology on U called as the Nano topology on U with respect to X. We call $(U,\tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets.

Definition 2.4 [4] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B=\{U,\ L_R(X),\ U_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5 [4] If $(U,\tau_R(X))$ is a Nano Topological space with respect to X where $X\subseteq U$ and if $A\subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by N int(A). Nano interior is the largest Nano open subset of A. The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A). It is the smallest Nano closed set containing A.

Definition 2.6 [1] Let $(U,\tau_R(X))$ be a Nano Topological space. A subset A of $(U,\tau_R(X))$ is called Nano generalized closed set if $Ncl(A)\subseteq V$ where $A\subseteq V$ and V is Nano open.

Definition 2.7 [9] Let $(U,\tau_R(X))$ be a Nano Topological space. A subset A of $(U,\tau_R(X))$ is called Nano weakly generalized closed (briefly Nwg-closed) set if $Ncl(N\operatorname{int}(A))\subseteq V$ where $A\subseteq V$ and V is Nano open.

Definition 2.8 [12] A function $f:X\to Y$ is called weakly generalized closed map if f(A) is weakly generalized closed set in Y for every closed set A in X.

Definition 2.9 Let $(U,\tau_R(X))$ and $(V,\tau_R{}'(Y))$ be Nano topological spaces. Then $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$

(i) Nano continuous [5] if f^{-1}(A) is Nano- open in U for every Nano-open set A in V.
(ii) Nano g-continuous [2] if f^{-1}(A) is Ng- open in U for every Nano-open set A in V.

Definition 2.10 [10] The map $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$ is Nwg-continuous on U if the inverse image of every Nano closed set in V is Nano weakly generalized closed in U.

Definition 2.11 [5] A function $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$ is a Nano open if the image of every nano open set in U is Nano open in V.

A function $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$ is a Nano closed if the image of every nano closed set in U is Nano closed in V.

Definition 2.12 [2] A function $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$ is a Nano g-open if the image of every nano g open set in U is Nano g open in V. A function $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$ is a Nano g- closed if the image of every nano g closed set in U is Nano closed in V.

Definition 2.13[5] A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be Nano-homeomorphism if f is 1-1 and onto, Nano-continuous and Nano-open.

Definition 2.14 [2] A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be Nano g-homeomorphism if f is 1-1 and onto, g-continuous and g-open.

Theorem 2.15 [9] If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nwg-irresolute function then f is Nwg- continuous function.

III. Nano Weakly Generalized Closed Map

In this section a new function in Nano Topological space called Nwg-closed map is introduced and few of its characterization are studied.

Definition 3.1

The map $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$ is Nwg-closed map if the image of every Nano closed set in $(U,\tau_R(X))$ is Nwg - closed in $(V,\tau_R{}'(Y))$.

Example 3.2

Let $U=V=\{0,1,2,3,4,5,6,7,8,9\}$, $X=\{1,5,7\}$, $R=\{x/x-y \text{ is divisible by } 2 \text{ and } x,y\in U\}$ $U/R=\{\{0,2,4,6,8\},\{1,3,5,7,9\}\}$ Then the Nano topology is $\tau_R(X)=\{U,\phi,\{0,2,4,6,8\},\{1,3,5,7,9\}\}$. Let $R'=\{x/x-y \text{ is divisible by } 3 \text{ and } x,y\in V\}$ $V/R'=\{\{0,3,6,9\},\{1,4,7\},\{2,5,8\}\}$, $Y=\{0,1,4,7\}$, $\tau_{R'}(Y)=\{V,\phi,\{0,1,3,4,6,7,9\},\{1,4,7\},\{0,3,6,9\}\}$. Let $f:(U,\tau_R(X))\to(V,\tau_R(Y))$ be an identity map then f is Nwg-closed map.

Remark 3.3

The composition of two Nwg-closed maps need not be Nwg-closed map as seen from the following example.

Example 3.4

Let $U=V=W=\{0,1,2,3,4,5,6,7,8,9\}$, $X=\{1,5,7\}$, $R=\{x/x-y \text{ is divisible by } 2 \text{ and } x,y\in U\}$ $U/R=\{\{0,2,4,6,8\},\{1,3,5,7,9\}\}$. Then the Nano topology is $\tau_R(X)=\{U,\varphi,\{0,2,4,6,8\},\{1,3,5,7,9\}\}$.

Let $R'=\{x/x-y \text{ is divisible by } 9 \text{ and } x,y\in V\}$ $V/R'=\{\{0,9\}\}$, $Y=\{3,8\}$, then $\tau_{R'}(Y)=\{V,\phi\}$

Let $R''=\{x/x-y \text{ is divisible by } 4 \text{ and } x,y\in W\}$, $W/R'=\{\{0,3,6,9\},\{1,4,7\},\{2,5,8\}\}$, $Z=\{0,1,4,7\}$, $\tau_{R'}(Z)=\{W,\varphi,\{1,3,5,7,9\},\{3,7\},\{5,7,9\}\}$.

Let $f:(U,\tau_R(X))\to(V,\tau_R(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R'}(Z))$ be an identity maps then f and g are Nwg-closed maps. But their composition is not Nwg-closed map since image of the Nano closed set $\{1,3,5,7,9\}$ is not Nwg - closed in $(W,\tau_R(Z))$.

Remark 3.5

Image of a Nwg-closed set need not be Nwg-closed set under a Nwg-closed map.

Example 3.6

Let $U = V = \{0,1,2,3,4,5,6,7,8,9\}$, $X = \{2,9\}$

$R = \{x / x - y \ \ is\ divisible\ by\ 5\ and\ x, y \in U\}$

$U / R = \{\{0,5\},\{1,6\},\{2,7\},\{3,8\},\{4,9\}\}$,

$\tau_R(X) = \{U, \phi, \{2,4,7,9\}, \{0,1,3,5,6,8\}\}$.

Let $R' = \{x / x - y \ \ is\ divisible\ by\ 6\ and\ x, y \in V\}$, $Y = \{1,2,8\}$ $U / R' = \{\{0,6\},\{1,7\},\{2,8\},\{3,9\}\}$ then the topology is $\tau_{R'}(Y) = \{V, \phi, \{2,8\}, \{1,7\}, \{1,2,7,8\}\}$. Let $f : (U, \tau_R(X)) \to (V, \tau_R{}'(Y))$ be an identity map. Then f is Nwg-closed map. Consider the set {2,7,8} which is Nwg-closed in $(U, \tau_R(X))$. But not Nwg-closed in $(V, \tau_R{}'(Y))$.

Theorem 3.7

Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a Nano closed map and $g : (V, \tau_{R'}(Y) \to (W, \tau_{R''}(Z))$ be a Nwg-closed map then their composition is Nwg-closed map.

Proof: Let A be any Nano closed set in $(U, \tau_R(X))$. Then $f(A)$ is Nano closed in $(V, \tau_{R'}(Y))$ and $(g \circ f)(A) = g(f(A))$ is Nwg closed since g is Nwg-closed map.Hence the composition is Nwg-closed map.

Remark: 3.8

If f is Nwg-closed map, g is Nano closed map then their composition need not be Nwg-closed map.

Theorem:3.9

A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nwg-closed if and only if for each subset A of V and for each Nano open set F containing $f^{-1}(A)$ there is a Nwg-open set B of V such that $A \subset V$ and $f^{-1}(V) \subset F$.

Proof:

Suppose f is Nwg-closed. Let A be a subset of V and F is an open set of U such that $f^{-1}(V) \subset F$. Then $B = V - f(U - F)$ is a Nwg-open set containing A such that $f^{-1}(V) \subset F$.

Conversely, suppose that C is a closed set of U. Then $f^{-1}(V - f(C)) \subset U - C$ and $U - C$ is Nano open. By hypothesis there is Nwg-open set B of V such that $V - f(C) \subset B$ and $f^{-1}(B) \subset U - C$.

Therefore $C \subset U - f^{-1}(B)$.

Hence $V - B \subset f(C) \subseteq f(U - f^{-1}(B)) \subset V - B$ which implies that $f(C) = V - B$. Since $V - B$ is Nwg-closed set $f(C)$ is Nwg-closed. Hence f is Nwg-closed.

Theorem 3.10

Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be any function and if $\tau_R(X) \neq \{U, \varphi\}$ and $\tau_{R'}(Y) = \{V, \varphi\}$. Then

(i) *f* is not Nano closed map.

(ii) f is Ng-closed map.

(iii) f is Nwg-closed map.

Proof of (i)

Since the only Nano closed sets in $(V, \tau_{R'}(Y))$ are $\{V, \varphi\}$, the image of Nano closed sets are not Nano Closed in $(V, \tau_{R'}(Y)$. Hence f is not Nano closed map.

Proof of (ii)

Let A be any Nano closed set in $(U, \tau_R(X))$. The Nano open set containing *f*(*A*) is *V*. Hence $Ncl(f(A)) \subset V$. (i.e.,) *f*(*A*) is Ng -closed. Therefore f is Ng-closed map.

Proof of (iii)

Let A be any Nano closed set in $(U, \tau_R(X))$. The Nano open set containing f (A) is V. Hence $Ncl(N\mathrm{int}(f(A))) \subset V$. (i.e.,) *f*(*A*) is Nwg closed. Therefore f is Nwg-closed map.

Theorem 3.11

Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be any function. Let *A* be a Nano closed set in $(U, \tau_R(X))$. Suppose if *V* is the only Nano open containing *f*(*A*), then f is Nwg-closed map.

Proof:

By assumption $f(A) \subset V$ where V is the only Nano open set containing *f*(*A*), then $Ncl(N\mathrm{int}(f(A))) \subset V$. Hence *f*(*A*) is Nwg-closed. Therefore f is Nwg-closed.

Theorem 3.12

Every Nano -closed map is a Nwg-closed map.

Proof:

Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a Nano closed map.Let *A* be a Nano closed set in the Nano topological space $(U, \tau_R(X))$. Then the image of A under the map f is Nano closed in the Nano topological space $(V, \tau_{R'}(Y))$. Since every Nano closed set is Nwg-closed set. $f(A)$ is Nwg-closed set. Hence *f* is Nwg-closed.

Remark 3.13

The converse of the above theorem need not be true as seen from the following example.

Example 3.14

In Example 3.2 *f* is Nwg-closed. But *f* is not Nano closed, since image $\{0,2,4,6,8\}, \{1,3,5,7,9\}$ are not Nano closed in $(V, \tau_R{}'(Y)$.

Theorem 3.15

Every Nano g-closed map is a Nwg-closed map not conversely.

Proof:

Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a Nano g-closed map. Let *A* be a Nano closed set in the Nano topological space $(U, \tau_R(X))$. Then the image of A under the map f is Nano g-closed in the Nano topological space $(V, \tau_{R'}(Y))$. Since every Nano g-closed set is Nwg-closed set. *f*(*A*) is Nwg-closed set. Hence f is Nwg-closed.

Remark: 3.16

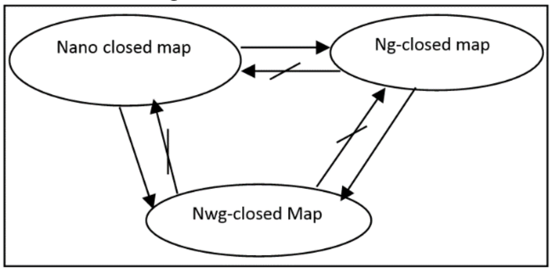
Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ be any function and if $f(A)\subset B, B$ is Nano open in $(V,\tau_R{}'(Y))$, $N\mathrm{int}(f(A))=\varphi, Ncl(f(A))\not\subset B$. Then f is Nwg-closed but not Ng –closed. It follows from the following example..

Example 3.17

Let $U=V=\{a,b,c,d\}$ with then the Nano topology is $\tau_R(X)=\{U,\varphi,\{b,c,d\}\}$ and $\tau_{R'}(Y)=\{U,\varphi,\{a,b,c\}\}$. Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is identity map. Then f is Nwg-closed. But not Nano g-closed, since, $Ncl(f\{a\})=Ncl(\{a\})=V\not\subset\{a,b,c\}$, $\{a\}$ is not Nano g closed in $(V,\tau_R{}'(Y))$

Remark: 3.18

From the above discussion and result, the relationship between the Nwg-closed map and the other closed maps are as bellow in the diagrams.



IV. Nano Weakly Generalized Open Map

In this section a weaker form of open map namely Nwg-open map is introduced and few of its properties are discussed.

Definition 4.1

The map $f:(U,\tau_R(X))\rightarrow(V,\tau_R{}'(Y))$ is Nwg-open map if the image of every Nano open set in $(U,\tau_R(X))$ is Nwg-open in $(V,\tau_R{}'(Y))$.

Example 4.2 Let $U=V=\{0,1,2,3,4,5,6,7,8,9\}$, $X=\{2,9\}$, $R=\{x/x-y \text{ is divisible by } 5 \text{ and } x,y\in U\}$

$U/R=U/R'=\{\{0,5\},\{1,6\},\{2,7\},\{3,8\},\{4,9\}\}$ then $\tau_R(X)=\{U,\varphi,\{2,4,7,9\},\{0,1,3,5,6,8\}\}$. Let $R'=\{x/x-y \text{ is divisible by } 9 \text{ and } x,y\in V\}$, $V/R'=\{\{0,9\}\}$, $Y=\{3,8\}$, $\tau_{R'}(Y)=\{V,\varphi\}$. Let f be an identity map from the topological space U to V then f is Nwg-Open map.

Remark 4.3

The composition of two Nwg-open maps need not be Nwg-open map as seen from the following example.

Example 4.4

Let $U=V=W=\{0,1,2,3,4,5,6,7,8,9\}$ $X=\{1,5,7\}$ $R=\{x/x-y \text{ is divisible by } 2 \text{ and } x,y\in U\}$ $U/R=\{\{0,2,4,6,8\},\{1,3,5,7,9\}\}$, then topology is $\tau_R(X)=\{U,\varphi,\{0,2,4,6,8\},\{1,3,5,7,9\}\}$.

Let $R'=\{x/x-y \text{ is divisible by } 9 \text{ and } x,y\in V\}$

$V/R'=\{\{0,9\}\}$, $Y=\{3,8\}$, $\tau_{R'}(Y)=\{V,\varphi\}$.

Let $R''=\{x/x-y \text{ is divisible by } 3 \text{ and } x,y\in W\}$,

$W/R''=\{\{0,3,6,9\},\{1,4,7\},\{2,5,8\}\}$, $Z=\{0,1,4,7\}$,

$\tau_{R''}(Z)=\{W,\varphi,\{1,4,7\},\{0,1,3,4,6,7,9\},\{0,3,6,9\}\}$. Let $f:(U,\tau_R(X))\rightarrow(V,\tau_R{}'(Y))$ and $g:(V,\tau_{R'}(Y))\rightarrow(W,\tau_{R''}{}'(Z))$ be an identity maps then f and g are Nwg-open maps. But their composition is not Nwg-open map since image of the Nano open set $\{0,2,4,6,8\}$ is not Nwg-open in $(W,\tau_R(Z))$.

Remark: 4.5

Image of a Nwg-open set need not be Nwg-open set under a Nwg-open map.

Example 4.6

Let $U=V=\{0,1,2,3,4,5,6,7,8,9\}$, $X=\{1,4,8\}$, $R=\{x/x-y \text{ is divisible by } 7 \text{ and } x,y\in U\}$

$U/R=\{\{0,7\},\{1,8\},\{2,9\}\}$ then $\tau_R(X)=\{U,\varphi,\{1,8\}\}$.

Let, $R'=\{x/x-y \text{ is divisible by } 6 \text{ and } x,y\in V\}$,

$Y=\{2,8\}$ $U/R'=\{\{0,6\},\{1,7\},\{2,8\},\{3,9\}\}$ then $\tau_{R'}(Y)=\{V,\varphi,\{2,8\},\{1,7\},\{1,2,7,8\}\}$. Let $f:(U,\tau_R(X))\rightarrow(V,\tau_R{}'(Y))$ be an identity map. Then f is Nwg-open map but not Nwg-open, since the set $\{0,2,3,4,5,6,9\}$ is Nwg-open in $(U,\tau_R(X))$ but not Nwg-open in $(V,\tau_R{}'(Y))$.

Theorem 4.7

Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ be any function and if $\tau_R(X)\neq\{U,\varphi\}$ and $\tau_{R'}(Y)=\{V,\varphi\}$. Then

(i) f is not Nano open map.
(ii) f is Ng-open map.
(iii) f is Nwg-open map.

Proof of (i)

Since the only Nano open sets in $(V,\tau_{R'}(Y)$ are $\{V,\varphi\}$, the image of Nano open sets are not Nano open in $(V,\tau_{R'}(Y)$. Hence f is not Nano open map.

Proof of (ii)

Let A be any Nano open set in $(U,\tau_R(X))$. The Nano open set containing $(f(A))^C$ is V. Hence $Ncl((f(A))^C)\subset V$. $f(A))^C$ is Ng-closed. $f(A)$ is Ng -open. Therefore f is Ng-open map.

Proof of (iii)

Let A be any Nano open set in $(U,\tau_R(X))$. The Nano open set containing $(f(A))^C$ is V.

Hence $Ncl(N\operatorname{int}((f(A))^C)) \subset V$. $(f(A))^C$ is Nwg-closed. *f(A)* is Nwg-open. Therefore f is Nwg-open map.

Theorem 4.8

Every Nano -open map is a Nwg-open map, but not conversely.

Proof:

Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be a Nano open map. Let A be a Nano open set in the Nano topological space $(U,\tau_R(X))$. Then the image of A under the map *f* is Nano open in the Nano topological space $(V,\tau_{R'}(Y))$. Since every Nano open set is Nwg-open set. *f(A)* is Nwg-open set. Hence *f* is Nwg-open map.

Example 4.9

In Example 3.2 *f* is Nwg-Open. But *f* is not Nano open, Since image $\{0,2,4,6,8\},\{1,3,5,7,9\}$ are not Nano open in $(V,\tau_R{}'(Y)$.

Theorem 4.10

Every Nano g-open map is a Nwg-open map. The converse need not be true.

Proof:

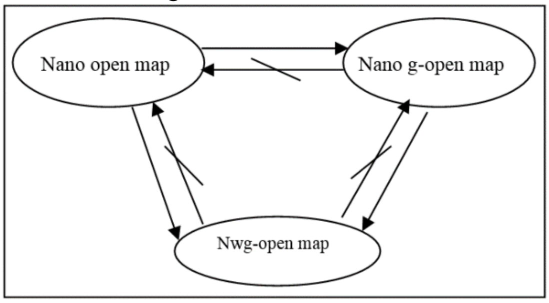
Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be a Nano g-open map. Let A be a Nano open set in the Nano topological space $(U,\tau_R(X))$. Then the image of A under the map f is Nano g-open in the Nano topological space $(V,\tau_{R'}(Y))$. Since every Nano g-open set is Nwg-open set. $f(A)$ is Nwg-open set. Hence f is Nwg-open map.

Example 4.11

In Example 3.17, f is Nwg-open map, but not Ng-closed map since the image of the Nano open set $\{b,c,d\}$ is not Ng-open set in V.

Remark 4.12

From the above discussion and result, the relationship between the Nwg-closed map and the other open maps are as bellow in the diagrams.



V. Nano Weakly Generalized Homeomorphism and Nano Weakly Generalized* Homeomorphism

This section is to define Nwg-homeomorphism and Nwg*-homeomorphism in Nano topological spaces

Definition 5.1

A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is said to be Nwg-homeomorphism if f is 1-1 and onto, Nwg-continuous and Nwg-open.

Remark 5.2

A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is said to be Nwg-homeomorphism if is 1-1 and onto, Nwg-continuous and Nwg-closed.

Theorem: 5.3

Every Nano homeomorphism is Nwg-homeomorphism.

Proof:

Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be a Nano homeomorphism. Then by definition it is 1-1 and onto, Nano continuous and Nano open map. Since every Nano continuous is Nwg-continuous and every Nano open map is Nwg –open map *f* is Nwg-homeomorphism.

Remark 5.4

The converse of above need not be true.

Example 5.5

In Example 3.2 *f* is Nwg-homeomorphism. But *f* is not Nano-homeomorphism.

Theorem 5.6

Every Nano g-homeomorphism is Nwg-homeomorphism.

Proof:

The proof of the theorem follows from the fact that every Nano g-continuous function is Nwg-continuous and every Ng-open map is Nwg-open map.

Remark5.7

The converse of above need not be true.

Example 5.8

In Example 3.17 *f* is not g-homeomorphism but Nwg -homomorphism.

Theorem 5.9

Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be a bijective Nwg-continuous function. Then the following are equivalent.

(i) *f* is an Nwg-open map.
(ii) *f* is an Nwg-homeomorphism
(iii) *f* is an Nwg-closed map

Proof:

$(i) \to (ii)$

By the assumption, and (i) *f* is bijective, Nwg-continuous, Nwg-open map. By definition of Nwg-homeomorphism f is Nwg-homeomorphism.

$(ii) \to (iii)$

Since *f* is Nwg-homeomorphism, it is 1-1 and onto, Nwg-continuous and Nwg-open map. Let A be a Nano closed set in $(U,\tau_R(X))$. Then $f(U-A)$ is Nwg- open in $(V,\tau_{R'}(Y))$. $f(U-A) = f(U) - f(A) = V - f(A)$ is Nwg-open. Hence f(A) is Nano closed set in $(V,\tau_{R'}(Y)$.

$(iii) \to (i)$

Let A be Nano open set in $U,\tau_R(X)$. Then $f(U-A)$ is Nwg-closed in $(V,\tau_{R'}(Y)$. (i.e.) *f(A)* is Nwg- open in $(V,\tau_{R'}(Y)$. Therefore *f* is Nwg-open map.

Definition 5.10

A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is said to be Nwg*-homeomorphism if *f* and f^{-1} both are Nwg-irresolute.

Remark 5.11

If a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ satisfies the following conditions then it is Nwg*-homeomorphism
(i) *f* is 1-1 and onto (ii) *f* is Nwg-irresolute function
(iii) *f* is Nano closed (Nano open) map

Theorem 5.12

Every Nano Nwg*- homeomorphism is Nwg-homeomorphism.

Proof: The proof of this theorem follows from the definition and by the theorem 2.15.

Remark 5.13

Converse of the above theorem need not be true, Since Nwg-continuous function need not be a Nwg-irresolute function.

Example 5.14

Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and $X=\{a,b\}$. Then $\tau_R(X)=\{U,\varphi,\{a\},\{b,d\},\{a,b,d\}\}$.

Let $V=\{p,q,r,s\}$ with $V/R'=\{\{p\},\{s\},\{q,r\}\}$ and $Y=\{p,r\}$. $\tau_R{}'(Y)=\{V,\varphi,\{p\},\{q,r\},\{p,q,r\}\}$.

Let $f:(U,\tau_R(X))\to(V,\tau_R{}'(Y))$. by $f(a)=q, f(b)=r,\ f(c)=s, f(d)=p$ then $f^{-1'}(\{p,q,r\})=\{a,b,d\}$, $f^{-1}(\{q,r\})=\{a,b\}$, $f^{-1}(\{p\})=\{d\}$ $f^{-1}(\varphi)=\varphi$ and $f^{-1}(V)=U$ where $\{a,b,d\}$, $\{a,b\}$, $\{d\}$, φ and U are Nwg-open sets in the Nano topological space $(U,\tau_R(X))$. Hence *f* is Nwg-continuous function.

The images of the open sets are $f(\{a\})=\{q\}$ $f(\{b,d\})=\{p,r\}$, $f(\{a,b,d\})=\{p,q,r\}$ $f(\varphi)=\varphi$ and $f(U)=V$ respectively. Where $\{p,q,r\}$, $\{p,r\}$, $\{q\}$, φ and *U* are Nwg-open sets in the Nano topological space $(V,\tau_{R'}(Y))$. Hence *f* is Nwg-open function. Hence *f* is Nwg-homeomorphism but *f* is not Nwg–irresolute since the set $f^{-1}(\{q\})=\{a\}$ is not Nwg-closed set in *U*. Hence *f* is not Nwg* homeomorphism.

VI. Conclusion

In this paper we introduced a new class of closed and open map in Nano Topological Spaces Called Nano weakly generalized closed map and Nano weakly generalized open map respectively. These maps are introduced using a weaker form of a generalized closed sets in Nano Topological spaces. A comparative study has been made with the existing closed and open maps. Suitable examples are given to support the study. Various characterizations for both Nano weakly generalized closed map and Nano weakly generalized open map have been analysed. In section V the investigation is made on Nano weakly generalized homeomorphism and an equivalent condition has been established. Also we discussed the properties of Nano weakly generalized*-homeomorphism.

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