Super Quantization of a Cantorian Electromagnetic Field and the Cosmic Dark Energy Density of the Universe

Mohamed S. ElNaschie
Distinguished Professor, Dept. of Physics, Faculty of Science, University of Alexandria, Alexandria, Egypt

Abstract – E-infinity Cantorian spacetime theory is a naturally golden mean quantized universe within which the difference between field and spacetime is fairly blurred. In particular the theory reconciles the un-reconcilable, namely preserving the most important features of true discreteness as well as the cardinality of the continuum. Applying this extraordinary versatile theory to the corresponding electromagnetic field and its golden mean scaling, an equally extraordinarily simple methodology leads to the exact cosmic density of ordinary energy and its complimentary dark energy is found. We also give a hint to the possible connection between dark energy as presented here and the work of Nikolay Umov and Friedrich Hasenoehrl.

Keywords – Cosmic Dark Energy, Super Quantization, Cantorian Electromagnetism, E-Infinity Theory, Penrose Universe, Golden Mean Scaling, Noncommutative Geometry, Zero Set Quantum Particle, Empty Set Quantum Wave, Electromagnetic Mass, Nikolay Umov, Friedrich Hasenoehrl.

I. INTRODUCTION

The question of why E-infinity Cantorian spacetime [1-15] is more than ‘unreasonably’ effective is multifaceted and deep but could probably be reduced to the miraculous but rational mathematical properties of its building blocks, namely the elementary Mauldin-Williams random Cantor sets [3, 4].

The present work starts by looking in some detail at the various geometrical and topological features of random Cantor sets which unite the un-uniteable and leads to a spacetime which is both discrete and naturally quantized yet shares with the continuum one of its most important aspects, namely its cardinality [1][3-5]. Equally important as much as surprising is the fact that a uniformly random geometrical construction should single out the epityome of harmonic order, namely the golden mean as its topological invariant which is the Hausdorff dimension of a topologically zero dimensional thin Cantor set of zero measure embedded into another positive measure fat but empty Cantor set [3-7]. Such superficially contradictory facts, which are never the less mathematically accurate offer undreamed of opportunities to solve paradoxical physical phenomena which are characteristic for the quantum world [9-31]. In fact following the path of E-infinity theory we do not need to write equations of motion explicitly and consequently we need not write down a Lagrangian. It is one of the undreamed of simplifications of E-infinity theory that every time we scale down by multiplying with (1/ϕ) we are quasi differentiating and the reverse scaling using ϕ is quasi integrating [1-5]. That way the time honoured calculus becomes a simple scaling which is readily applicable to non-smooth and discontinuously behaving systems which could not be differentiated nor integrated in any classical sense [1-15].

The present work utilizes all the above in quantizing the electromagnetic field and uncovers its deep connection with the ordinary and dark energy sectors of the cosmos and we may start by looking at the vast web of golden mean scaling relations implicit by the golden mean super quantization of E-infinity Cantorian spacetime [3, 4, 6, 7]. In addition we speculate towards the end of this paper on possible connections with the pioneering work of NikolayUmov and Friedrich Hasenoehrl [29-31].

II. THE MAGIC OF AN ELEMENTARY CANTOR SET AND PENROSE FRACTAL UNIVERSE

The classical triadic Cantor set is well known from the theory of nonlinear dynamics being the backbone set of all fractal sets [3, 4]. The construction of this set was discussed in considerable details on many previous occasions [25] and it is sufficient to mention here that the only non-zero quantity is the Hausdorff dimension of this zero measure and zero topological dimension set is given by \( \lambda n^2 \approx n^3 \ 0.6309297536 \). Assuming uniform distribution, Mauldin and Williams [3] were able to show that an analogously randomly constructed version of the deterministic Cantor set has the golden mean \( \phi = (\sqrt{5} - 1)/2 \approx 0.6180339887 \) as its Hausdorff dimension [5, 6]. The difference between \( \phi \) and \( \lambda n^2 \) is clearly very small in quantitative terms, in fact not more than 2%. However in qualitative terms this minor difference makes a world of difference as we will see shortly because of the ‘unreasonable’ effectiveness of the golden mean number system as compared to any other computational number system what so ever.

Now a natural question arises, namely what is the higher dimensional generalization of \( D = (O, \phi) \) Cantor set? [5, 7, 13]. This is not the same question as that of embedding. It is obvious in the simple case of constructing \( D = (O, \phi) \) that it is embedded in a one topological dimension. On the other hand, generalizing \( D = (O, \phi) \) would mean looking for the Hausdorff dimensions corresponding to the positive as well as the negative dimension. Now this fundamental question is answered by the bijection formula of E-infinity theory which states that [3, 7, 10, 13].

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\[ d^{(0)} = \left( \frac{1}{\phi} \right)^{n-1} \]  
(1)

Thus for \( n = 0 \) one finds that
\[ d^{(0)} = \left( \frac{1}{\phi} \right)^{-1} = \phi \]  
(2)

which is just another economic notation for saying
\[ D(O) = (O, \phi) \]  
(3)

This is the zero set which is used in E-infinity theory to model the pre-quantum particle. Next for \( n = 1 \) one finds the normality set, namely \([3, 7, 8]\).
\[ d^{(1)} = \left( \frac{1}{\phi} \right)^{2-1} = 1 + \phi \]  
(4)

For two and three and four topological dimensions one finds
\[ d^{(2)} = \left( \frac{1}{\phi} \right)^{3-1} = 1 + \phi \]  
(5)

and
\[ d^{(3)} = \left( \frac{1}{\phi} \right)^{4-1} = 2 + \phi \]  
(6)

as well as the well known fractal version of Einstein’s spacetime dimension, namely
\[ d^{(4)} = \left( \frac{1}{\phi} \right)^{5-1} = 4 + \phi^3 \]  
(7)

This is the by now well known E-infinity core dimension which leads to the combined Einstein-Kaluza fractal spacetime dimension \( 5 + \phi^3 \) used extensively in various recent publications to quantify ordinary energy, dark matter energy and pure dark energy [9-22]. This point becomes even more interesting when we proceed towards negative dimensions. Thus the empty set \( D = -1 \) becomes something rather than nothing and plays a pivotal role as the Aether of the theory [9-12]. By contrast what is particularly relevant to the present paper is the fact that the bijection formula is the mathematics behind the geometry and topology of Sir. R. Penrose’s fractal universe which is the fundamental mathematical model for our actual universe and fully describes its holographic boundary in the sense of ‘tHooft and Susskind [18-20]. In turned out that the symmetry group of this universe is given by SL(2, 7) after adding a compactification replacing SU(2) by 16\( k \) where \( k \) is ‘tHooft’s renormalon. Noting that
\[ k = \phi^5 (1 - \phi^3) = 0.18033989 \]  
is twice the magnitude of Hardy’s probability of quantum entanglement, we see that we can write [18-20].
\[ \dim[SL(2,7) + SU(2)] = 336 + 3 \]  
(9)
\[ = 339 \]

in a transfinite accurate way as the holographic boundary dimension [18-20]
\[ \dim(H) = 336 + 16k \]  
(10)
\[ = 338.88543824 \]

That way one can deduce the inverse electromagnetic fine structure “constant” from the E8E8 exceptional symmetry group of the bulk after adding the pure gravity component for eight dimensions, i.e. the 20 components and find the integer approximation [18-20].
\[ [E8E8] = [336 + 3 + 20] \]  
(11)
\[ = 496 - [359] \]
\[ = 137 \]
\[ = \bar{a}_n \]

The corresponding exact transfinite result is thus [18-20]
\[ \bar{a}_n = (496 - k^2) - [336 + 16k + 20] \]  
(12)
\[ = 137 + k_n \]

where \( k_n = \phi^5 (1 - \phi^3) \) and \( \phi^5 \) is Hardy’s probability of quantum entanglement [13-15]. Now it is time to show how A. Connes noncommutative geometry is not only related but actually identical formulation to the above.

**III. NONCOMMUTATIVE GEOMETRY OF THE PENROSE-E-INFINITY FRACTAL UNIVERSE [3-5]**

The equivalence between von Neumann-Connes dimensional function [3-5]
\[ D = a + b\phi \]  
(13)

where \( a, b \in \mathbb{Z} \) and \( \phi = (\sqrt{5} - 1)/2 \) and the E-infinity bijection formula was discussed extensively in [3-5]. The far reaching conclusion for this fact is the reasons that we stress the importance of this section although we kept it short and urge the reader to study the afore mentioned referenced and the references therein [3-20].

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IV. ORDINARY AND DARK COSMIC ENERGY DENSITY FROM ELECTROMAGNETISM

We hinted in the present introduction that using the golden mean number system which arises in a natural way from postulating Penrose fractal tiling universe as the mathematical blue print of our actual cosmos is de facto a super quantization procedure where the adjective “super” is used to highlight the unique fact that we have a discrete theory with the cardinality of the continuum [1-15]. This is a unique strength of our theory not shared by any other theoretical formulation. In addition or as a consequence of the above, our theory does not have the achilles weak point of having to define a point in the classical way [21] because it is a pointless geometrical-topological theory as envisaged by von Neumann in his “continuous” geometry [9-15]. Now taking \( \vec{\alpha}_n = 137 + k_n \) seriously as the most fundamental physical energy dependent “constant” in high energy physics as well as a dimension of the corresponding electromagnetic field then we can combine the anomaly cancellation of superstring theory with the twelve dimensional “father” spacetime of C. Vafa to claim that the exact measure of ordinary energy density of the cosmos is given by the ratio of the said two dimensions, namely the modified anomaly cancellation condition.

\[
2\vec{\alpha}_n = [(2)(137 + k_n)] - 1 + 1 = 274.1640787
\]

as the largest possible dimension in this case and \([28]\)

\[D(FF) = 2(6 + k) = 12 + 2k\]

of the “fractal” father theory \([28]\) representing the smallest involved dimension. In other words we claim that the ordinary cosmic energy density is given by

\[
\gamma(O) = k(O) = D(FF) / (2\vec{\alpha}_n)
\]

\[= 12.36067966 / 274.1640787 = 0.04512576\]

An elementary examination of the ratio reveals that it is our old exact result found using a host of different theories and method showing that it is half of Hardy’s famous probability of quantum entanglement [1,2,4]. Thus we may write

\[D(FF) / [(2\vec{\alpha}_n - 1) + 1] = \phi^2 / 4\]

\[
\approx \frac{1}{22}
\]

It is clearly a trivial matter to assert that \((2\vec{\alpha}_n - 1) + 1 = 2\vec{\alpha}_n\) is nothing but the number of particles in a super symmetric complete standard model \([23]\). The preceding result would strike us in the first instant as the sort of miracle that should be distrusted in any serious philosophical-scientific frame of mind. However when we examine the number theoretical web of connections between the fundamental quantities of the theory and their golden mean scaling, then we realize that we have not been seeing the wood for the trees and nothing is more natural for nature than super quantization of the golden mean scaling of E-infinity theory. The rest of the present paper will be devoted to shedding light on this basic technique.

V. DISCOVERING THE GOLDEN MEAN HIGH ENERGY SCALING WEB OF CONNECTIONS

Let us start with the remarkable scaling of a Cooper pair inverse electromagnetic fine structure “constant” which as we know from E-infinity, produces the exact Heterotic string theory dimensional hierarchy in its transfinitiely correct form. Proceeding this way one finds from \((\vec{\alpha}_n / 2)(\phi_n)\) that \([4,5,10,25]\).

\[
\begin{align*}
\left(\frac{137.08202939}{2}\right)^2 & \rightarrow 26 + k \equiv 26 \\
\left(\frac{137.08202939}{2}\right)^3 & \rightarrow 26 + k \equiv 16 \quad (18) \\
\left(\frac{137.08202939}{2}\right)^5 & \rightarrow 10 = 10
\end{align*}
\]

On the other hand if we go on further from \(n = 4\) to \(n = 6\) we obtain the corresponding hierarchy of superstring theory, namely \([5]\) [18-20] [25].

\[
\begin{align*}
\left(\frac{137.08202939}{2}\right)^4 & \rightarrow 10 = 10 \\
\left(\frac{137.08202939}{2}\right)^5 & \rightarrow 6 + k \equiv 6 \quad (19) \\
\left(\frac{137.08202939}{2}\right)^6 & \equiv 4 - k \equiv 4
\end{align*}
\]

we see that \(D = 10\) is indeed the demarcation point and link between the bosonic strong interaction string theory of Nambu and Veneziano and the Green Schwarz superstring theory and that the link yielded the exact transfinite Heterotic string theory of Nobel Laureate David Gross and his Princeton quartet collaborators [18-20] together with ‘tHooft-Veltman-Wilson fractal spacetime \(D = 4 - k\) and ‘tHooft’s renomalon \(k = \phi (1 - \phi^2)\) which as we can see amounts to twice the value of Lucien Hardy’s exact probability of the quantum entanglement of two quantum particles, i.e. \(\phi^2\) [15]. The miraculous result becomes even more mind blowing when we realized that it agrees almost perfectly not only with the accurate Hardy’s quantum entanglement real experiment but also with painstaking cosmic measurements and observations regarding the cosmic ordinary and dark energy sector of our universe as attested by the COBE, WMAP and supernova project, all of which were awarded at the time of the Nobel Prize in physics or cosmology [22].

To appreciate our preceding comments we just take the ratio between the main topological invariant of Einstein spacetime taken at its smooth and continuous face value, namely \(D = 4\) to the fractal-like D found after subjecting it to the golden mean “scaling” super quantization, namely \(D = 4 - k\) one will be struck by finding the result to be identical to \(1 - (\phi^2 / 2) = 5\phi^2 / 2\) \(\equiv 95.5\%\), i.e. the cosmic dark energy density which cannot be measured directly because of quantum wave collapse [2, 5, 10][15-20]. We say strict in the full knowledge that it seems a logical and intuitively clear result supported by common sense that the ratio \(1 / (4 - k) \equiv 4.5\%\) should be a measure for the density...
of ordinary energy [2, 5, 10]. The problem is really a paradox in its own right because it is too nice to be true and too easily obtained to be easily believed. Luckily we reached the same result using more than a dozen different theories and methods so that the probability of error is practically zero [22-26].

VI. DISCUSSION

The golden mean super quantized E-infinity theory seems not only to be a very accurate high energy theory but far more than that, it seems to be the blueprint for the inner dynamics of nature at the building block level [25]. We have long known that the rational approximation of ordinary cosmic energy density is given by $1/22 \approx 4.5\%$ and as a result of this realization, Einstein’s maximal energy density $E = mc^2$ is in fact the sum to two parts [12] and should be understood as $E = \left(mc^2/22\right) + \left(21/22\right)mc^2$ [25]. Furthermore we also realized some time ago that this real energy factor $1/22$ could be understood as the ratio computed from two fundamental numbers of particle-like quantum states, namely 504 and 528 leading to [12]

$$\gamma(O) = \frac{(528 - 504)/528}{1/22} = 1/22$$

as explained in a great deal of detail for instance in Ref. [12]. On the other hand we also considered the scaling exponent $\lambda = 1/22$ involved in this result which is due to the trivial fact that [12]

$$24/528 = 12/264$$

$$= 6/132$$

$$= 3/66$$

It is also clear that the 132 in equation (21) is the dimension of an important moduli space compactification manifold in 4D string theory with toroidal compactification and 66 is the number found for instance in E. Witten’s five Brane in eleven dimensions model [24]. However the present analysis reveals that is 132 is also an approximation for $\varpi_0$ and that the exact result can be obtained when including all the transfinite correction terms so that we replace 6 by 6 plus ’tHooft’s renormalon and 132 by $\varpi_0 = 137 + k_0$. That way we recognize via the golden mean super quantization machinery that the exact density is given by a ratio revealing the electromagnetic origin of the dark energy sector because ordinary energy density is given by

$$\gamma(O) = \frac{6 + k}{137 + k_0} = \frac{\phi^2}{2}$$

and consequently the dark energy density of the cosmos is given by

$$\gamma(D) = 1 - \gamma(O)$$

$$= 1 - \left(\frac{\phi^2}{2}\right)$$

$$= \frac{5\phi^2}{2}$$

exactly as we found numerous times before but this time it is not only a correct result, it is a deeper result allowing us to see dark energy from yet another angle, namely that of electromagnetism.

VII. CONCLUSION

E-infinity, random Cantor sets, golden mean scaling and number systems are just different angles of looking at the language chosen by nature. The writer admits that only a few years ago such statements would have been regarded by him as esoteric or romantic nonsense or at a minimum half truths and over the top being exaggerated in a fashion he would not have approved of under any circumstances to say the least. Now the author must admit again that the situation has undergone a radical change. We now recognize the discovery of golden mean scaling as the discovery of nature’s super quantization where the superlative adjective “super” used here is not referring to super symmetry but to the unreasonable simplicity and effectiveness of the golden mean scaling as nature’s way to quantify without loosing the continuum and bringing classical, relativistic and orthodox quantum mechanics into a single theory, namely E-infinity theory which we feel was well anticipated by the work of Nobel Laureate J. Schwinger and some great representatives of the powerful Russian-Soviet school such as E. Fredkin and Nikolay Umov who seems to be the first to write a mass energy equation in the remarkable form $E = kmc^2$ anticipating cosmic dark energy as early as 1873 [29, 30]. Now this theory did not emerge from a vacuum. In fact E-infinity theory has borrowed generously from almost all previous theories such as von Neumann’s continuous and pointless geometry, Connes noncommutative geometry, Witten’s topological quantum field theory and topological string theory, D. Gross’ Heterotic superstring theory and many, many other physical, pure mathematical and applied theories particularly nonlinear dynamics, deterministic chaos and random fractals [19, 27]. In this context we must mention the remarkable 1904 result of F. Hasenohrl $E = (3/4)mc^2$ and the electromagnetic mass which may be related to the pure dark energy density of our cosmos [27, 29, 31].

In the present work we added more proof to what was established using dozens of methods, namely that $E = mc^2$ is beyond any reasonable doubt the correct maximal energy density in the universe but we need from now on to qualify this result by stating clearly that in this form the formula is the final sum of three components and that at present only the ordinary energy component can be measured in a direct way. We showed furthermore that this density is deeply linked to inverse electromagnetic fine structure constant $\varpi_0 = 137 + k_0$ [27, 29, 31].

This conclusion does not contradict previous ones that attribute ordinary energy to the energy of the classical photon. In fact it is the hallmark of a beautiful universe like ours that there are many different ways of looking at the same phenomena but ends with essentially the very same result. In this context we are reminded of the words of Niels Bohr “only in the abundance of things we can hope to achieve clarity”. In the original German words of the great Danish physicist and philosopher “Nur in der fülle liegt die Klarheit”. Finally we should mention that for an in depth reading around the wider implications of the present theory,
we recommend Ref. [1] as well as Refs. [32] and [33].

REFERENCES


AUTHOR’S PROFILE

Professor M.S. El Naschie was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his Dr.Dipl-ing diploma, equivalent to a Master’s degree in Chartered Structural Engineering. After that he moved to the UK where he enrolled as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt.

Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work is cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor. He is the creator of E-infinity theory, which is a physical theory based on random Cantor sets and can be applied to macro, micro and mesoscopic systems.

Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 74 and his i-10 index is 755 according to Google Scholar Citation.