International Journal of Innovation in Science and Mathematics
Volume 6, Issue 3, ISSN (Online): 2347–9051

Total Volume Flux from Mass Conservation in Equations of Motion on a Disk / Plate Surface

Steve Anglin, Sc.M., Ph.D.(h.c.)
Corresponding author email id: stevemanglin@gmail.com

Date of publication (dd/mm/yyyy): 05/05/2018

Abstract **– We look at mass conservation in equations of motion on a disk/plate surface. u/v, u/r, w are functions of z where (u, v, w) are velocity components of r, theta and z or (r, theta, z) in cylindrical coordinate system with u/r = 0. Also, pressure p is defined as p over rho = nu times w'(z) – ½(w^2) + some function F where F = F(r) = const. So, this can imply that we let p = const, then p trends towards zero or p → 0 in the equations of motion below for r and theta. Again, there is mass conservation which can lead to dimensionless equations of motion where with boundary conditions, we get the Reynolds number and get the total volume flux outward across cylindrical surface.**

Keywords **– Equations of Motion, Mass Conservation, Flux, Reynolds, Dimensionless, PDE, Cylinder, Disk, Surface, Volume, Fluid Dynamics, Applied Math, Differential Equation.**

We look at mass conservation in equations of motion on a disk/plate surface. u/v, u/r, w are functions of z where (u, v, w) are velocity components of r, theta and z or (r, theta, z) in cylindrical coordinate system with u/r = 0.

Also, pressure p is defined as p over rho = nu times w'(z) – ½(w^2) + some function F where F = F(r) = const.

$$\frac{p}{\rho} = \nu\frac{dw}{dz} - \frac{1}{2}w^2 + F$$

So, this can imply that we let p = const, then p trends towards zero or p → 0 in the equations of motion below for r and theta such that we have the following

$$(\frac{u}{r})^2 + w\frac{d(u/r)}{dz} - (\frac{v}{r})^2 = \nu\frac{d^2(u/r)}{dz^2}$$

$$\frac{2uv}{r^2} + w\frac{d(u/r)}{dz} = \nu\frac{d^2(u/r)}{dz^2}$$

Again, there is mass conservation which can lead to dimensionless equations of motion where with boundary conditions, we get the Reynolds number and get the total volume flux outward across cylindrical surface.

So, we start with this, with boundary conditions:

$$\frac{2u}{r} + \frac{dw}{dz} = 0$$

$$u = w = 0, v = u_\theta = \Omega r, z = 0$$

$$u \to 0, v \to 0, z \to \infty$$

$$\Rightarrow \frac{u}{r} = \frac{v}{r} = \Omega r g(\xi) r, w = (\nu\Omega)^{1/2}h(\xi)$$

$$z = (\frac{\nu}{\Omega})^{1/2}\xi, \xi = z(\frac{\Omega}{\nu})^{1/2}$$

The above can lead to dimensionless equations of motion:

$$f^2 + hf' - g^2 = f'', 2fg + hg' = g'', 2f + h' = 0$$

$$f = (-1/2)h'$$

$$-\frac{1}{2}h'^2 + h(-\frac{1}{2}h')' - g^2 = -\frac{1}{2}h'''$$

$$-\frac{1}{2}h'^2 + h(-\frac{1}{2}h')' - g^2 = -\frac{1}{2}h'''$$

$$2(-\frac{1}{2}h')g + hg' = g'', 2(-\frac{1}{2}h') + h' = 0$$

$$\Rightarrow \frac{1}{4}h'^2 - \frac{1}{2}hh'' - g^2 = -\frac{1}{2}h'''$$

$$-gh' + g'h = g''$$

With following Boundary Conditions*:*

$$h = h' = 0, g = 1, at\xi = 0, h' \to 0$$

$$g \to 0, \xi \to \infty. \Rightarrow h \to -.088$$

$$u_z \to -0.89(\nu\Omega)^{1/2}, z \to \infty$$

And, this Reynold's Number:

$$R = \Omega L^2/\nu$$

So, total volume flux outward across cylinder surface is the following:

$$\therefore F_{out} \sim 0.89\pi r^2(\nu\Omega)^{1/2}.$$

References

[1] Arnoldo, "Conservation of Mass (Volume)", Lecture Notes, ESSIE, University of Florida (2008).

[2] Batchelor, *an Introduction to Fluid Dynamics*, Cambridge University Press (2000).

[3] Brady & Durlofsky, Journal of Fluid Mechanics 175, pp. 363-394.
[4] Erich, Fluidmechanik Band 2, 4. Auflage, Springer Verlag, 1996, p. 178-179.
[5] Kundu, *Fluid Mechanics,* Elsevier (2003).
[6] Landau, *Fluid Mechanics*, Pergamon (1959).
[7] Maxey, Lecture Notes of AM242 Fluid Dynamics II, Brown University (2002).
[8] McDonough, "Lectures in Elementary Fluid Dynamics," University of Kentucky (2009).
[9] Rodel & Szerl, Physics F.9 (6), pp. 1650-1997.
[10] Tryggvason" The Equations Governing Fluid Motion", Computational Fluid Dynamics course, Notre Dame University (2011).

AUTHOR'S PROFILE

Steve Anglin, Sc.M, Ph.D. (h.c.) is an applied mathematician, a member in The Society of Industrial and Applied Mathematics, and a former visiting lecturer at Case Western Reserve University and Saint Leo University. He received his Master of Science in Applied Mathematics from Brown University of The Ivy League and Hon. Doctorate from Trinity College.