

On the Fractal Counterpart of C. Vafa's Twelve-Dimensional F-theory and the A. Schoenberg Twelve-Tone Music Implicit in the Standard Model of High Energy Elementary Particles

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Abstract – We report in this short letter on what we think is a major finding which could remove forever the schism that exists between Science and Art in an unprecedented way. In particular we believe that we were able to uncover an entire spectrum of scale invariant fractal Cantorian spacetime manifolds starting from Einstein's $D = 4$ spacetime and continuing via Kaluza-Klein $D = 5$, Superstring $D = 10$, Witten's M-theory $D = 11$ and Vafa's F-theory $D = 12$ to reach some tantalizing connections to the standard model of high energy particle physics as well as Arnold Schoenberg's twelve tone music. Our quantitative mathematical analysis depends in a crucial way on the golden mean number system of E-infinity theory which implies not only a real possibility of a theory of everything unifying all fundamental forces, but goes also far beyond that, suggesting a unification of facts and values i.e. Science and Art, encompassing music, poetry and philosophy.

Keywords – Pythagoras Cosmic Music, Number Theory in Physics, Twelve Tone, Golden Mean Harmony, Hardy Quantum Entanglement, Witten's M-theory, Vafa's F-theory, E-infinity Theory.

I. SCIENTIFIC FACTS AND HUMANISTIC VALUES MATHEMATICALLY UNIFIED. A SHORT SUMMARY

In short the present letter to the Editor is effectively a concise report on the final results of our decades long efforts to unify all the fundamental interactions of nature using the E-infinity theory methodology [1-35] which is based on the "pointless" geometry of Von Neumann [34, 36, 37] and the dimensional function which A. Connes devised to describe Penrose's fractal universe [19, 34, 37, 38, 39].

$$D = a + b\phi \tag{1}$$

where $a, b \in \mathbb{Z}$ and $\phi = (\sqrt{5} - 1) / 2$.

Continuing along this line of scientific interrogation, it became evident that much of the limitations inherent in Einstein's relativity theory can be removed by making Einstein's spacetime strictly scale invariant and this leads us to the self-similar Hausdorff dimension of a self-similar spacetime manifold $D = 4 + \phi^3$ [1, 3, 5, 6]. Subsequently we found that the same ideas can be directly applied to Witten's $D = 11$ and lead us directly to a fractal version of M-theory with $D = 11 + \phi^5$ as can be seen in the attached figures and charts [see chart 1 to 3 plus tables 1 to 3] as well as the relevant references [5, 22, 40, 41]. Furthermore, it was subsequently noticed that if a topological photon is put into the space $D = 11 + \phi^5$ of the fractal-Cantorian M-theory [5, 40, 41], then one finds that $D = \sqrt{\bar{\alpha}_0}$ where $\bar{\alpha}_0 = 137 + k_0$ is the inverse electromagnetic fine structure constant and $K_0 = \phi^5(1 - \phi^5)$ [5, 401, 42]. On the other hand $\sqrt{\bar{\alpha}_0} = 11.7082039$ is the exact number of bosons in the

standard model when we use fractal logic and fuzzy-fractal counting of the 12 messenger particles and arrive at the superficially absurd equation namely [41, 42],

$$12 = 14 = 11.7082039325 \tag{2}$$

Needless to say, the deep meaning of equation number (2) could be understood with fuzzy-fractal logic only [10, 32, 42]. Having made this realization and resolved the paradox which is not a real paradox but rather a super reality obeying super non classical logic [10, 33, 42], we were struck by yet another surprising discovery namely that Vafa's 12 dimension F-theory possesses a fractal version with fractal dimension equal to:

$$D(F) = 12 - 2\phi^4 \tag{3}$$

where ϕ^4 is the topological Unruh temperature [24, 28, 32]. On the other hand and again to our own great surprise this value is exactly equal to $\sqrt{\bar{\alpha}_0}$ and correspond in the same time to a fractal variant of F-theory spacetime. That means, the number theory center of our brain and logic tell us that:

$$D(F) = \sqrt{\bar{\alpha}_0} = 11.7082039325 \tag{4}$$

In other words, the fractal F-theory is in fact the space not only where the complete fractal standard model, including the Higgs and the graviton, lives but it is actually topologically identical, from a number theoretical viewpoint, to the standard model itself. Furthermore, looking at the 12-tone music "harmony" of A. Schoenberg [43], one is almost compelled to call it "a fractal music" and it is hard for us to overlook its connection to Vafa's F-theory on several counts [44, 45]. The point is that $\phi = (\sqrt{5} - 1)/2$ is a composition principle known, since the design of the pyramids of Egypt [46, 47], to play a major role in visual art as well as music [31, 43].

At the same time ϕ is the basis of the work of A. Connes' non-commutative geometry [39] of Penrose's fractal tiling universe as well as being the exact solution to Hardy's celebrated quantum entanglement probability [4,15].

Having reached these conclusions, it is impossible not to speculate that the schism between hard core science and high level visual art as well as poetry and music is an illusion and a historic misunderstanding and that facts and values meet in the golden mean based deep reality and truth of Nature which is in turn Spinoza's God [31]. Interestingly, when $D = 14$ is formerly considered as the complete number of an integer counting of the standard model messenger bosons obtained by adding to the 12 generators of the combined Lie symmetry groups $U(1) SU(2) SU(3)$ the Higgs and the graviton i.e. $12 + 2 = 14$, then the fractal version of our present theory using the same previous logic [10, 33, 42] would lead to:

$$D = 14 - 2\phi^4 = 13.702803935 = (\bar{\alpha}_0) / 10 \tag{5}$$

As incredible as it may be, we have retried yet again Wolfgang Pauli's magical prime number i.e. 137 with a golden mean k_0 irrational tail, made up of Hardy's quantum entanglement ϕ^5 and disentanglement $1 - \phi^5$ as can be deduced from $k_0 = \phi^5 (1 - \phi^5) = 0.082039325$ [4-15]. For an introduction to earlier and subsequent work the reader is referred to the relevant literature [1, 2, 5] and [46-50].

II. CLOSING REMARKS - SUPERSTRING FRACTAL MUSIC

The reader may be at least mildly surprised that we have already paved the way to a complete unification of all fundamental interactions including gravity via electromagnetism [34, 35]. However he or she will find out that we have done much more than that thanks to the fractal-fuzzy golden mean number system [32, 41], as evident from the suggested unification of science and art in the preceding sense stated above [31, 32].

Nevertheless the reader may have noticed that we did not tackle $D = 10$ super strings and super gravity particularly $D = (10/2) = 5$ where $D = 5$ is the dimension of Kaluza-Klein's fermionic spacetime which is fundamental and indispensable in understanding bright and dark energy [28, 42] (see also table 3).

On the other hand, there is a method and meaning behind the ensuing mad procedure linking $D = 10$ and $D = 5 + \phi^3$. The point is that we can regard the union of two $5 + \phi^3$ as super space leading to:

$$D = (2) (5 + \phi^3) = 10.47213595 \tag{6}$$

Let us put a topological photon in this space and we will immediately find that:

$$\begin{aligned} D &= 10.47213595 + \phi = 11 + \phi^5 \\ &= \text{Witten's fractal M space} \end{aligned} \tag{7}$$

From the preceding elementary calculation, we see how $D = 10$ and $D = 11 + \phi^5$ are all interlinked as a real spectrum of a fractal universe made of Hausdorff and topological dimensions [33, 41], [45-49]. With that we rest our case and hope that we have presented it in an acceptable way and in particular that the reader has benefitted from the charts and tables contained in this letter which hopefully will result in a deep understanding of the basic structure of E-infinity theory and the present basic message of this letter.

III. GENERAL CONCLUSION

The main thrust and most important conclusion of the present work is, in our opinion, the somewhat belated realization of the Author, if not of the entire scientific community worldwide, that the Pythagoreans got it right and miraculously so in posting the entire cosmos as unfolding from the musical harmony of Number Theory. How were these very early pioneers, who studied the Egyptian and Persian cultures in depth, able to come as far as this is still a mystery even to the present author. But then what is life without such wonderful revelations that are there for us to unravel and research as well as to uncover and admire. In short the music of numbers is similar to the popular song about love: "It makes the world go round".

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AUTHOR'S PROFILE



Professor **M.S. El Naschie** was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl.-Ing) diploma, equivalent to a Master's degree and Chartered Structural Engineering. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt. Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work in cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor. Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 79, his i-10 index is 779 and his citations according to Google Scholar is 34336.

CHART (1)

A chart explaining the logic and mathematic combining A. Connes' non-commutative geometry with E-Infinity which leads to an exact calculation of the "ordinary" bright energy and cosmic dark energy density of the universe as well as explain how Kaluza-Klein theory leads to the scale invariant Einstein spacetime dimension $D = 4 + \phi^3$ where ϕ is the golden mean $\phi = (\sqrt{5} - 1) / 2$

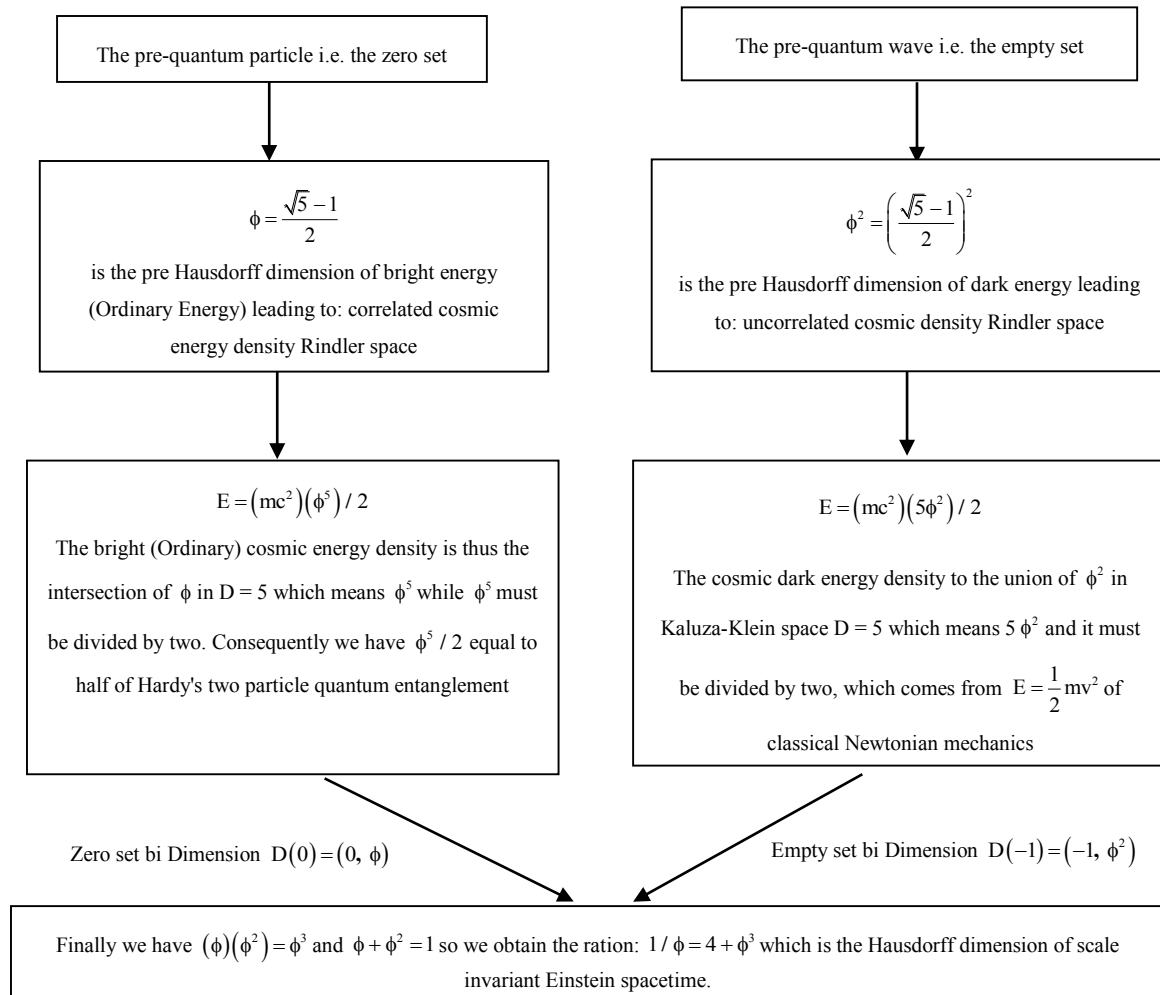
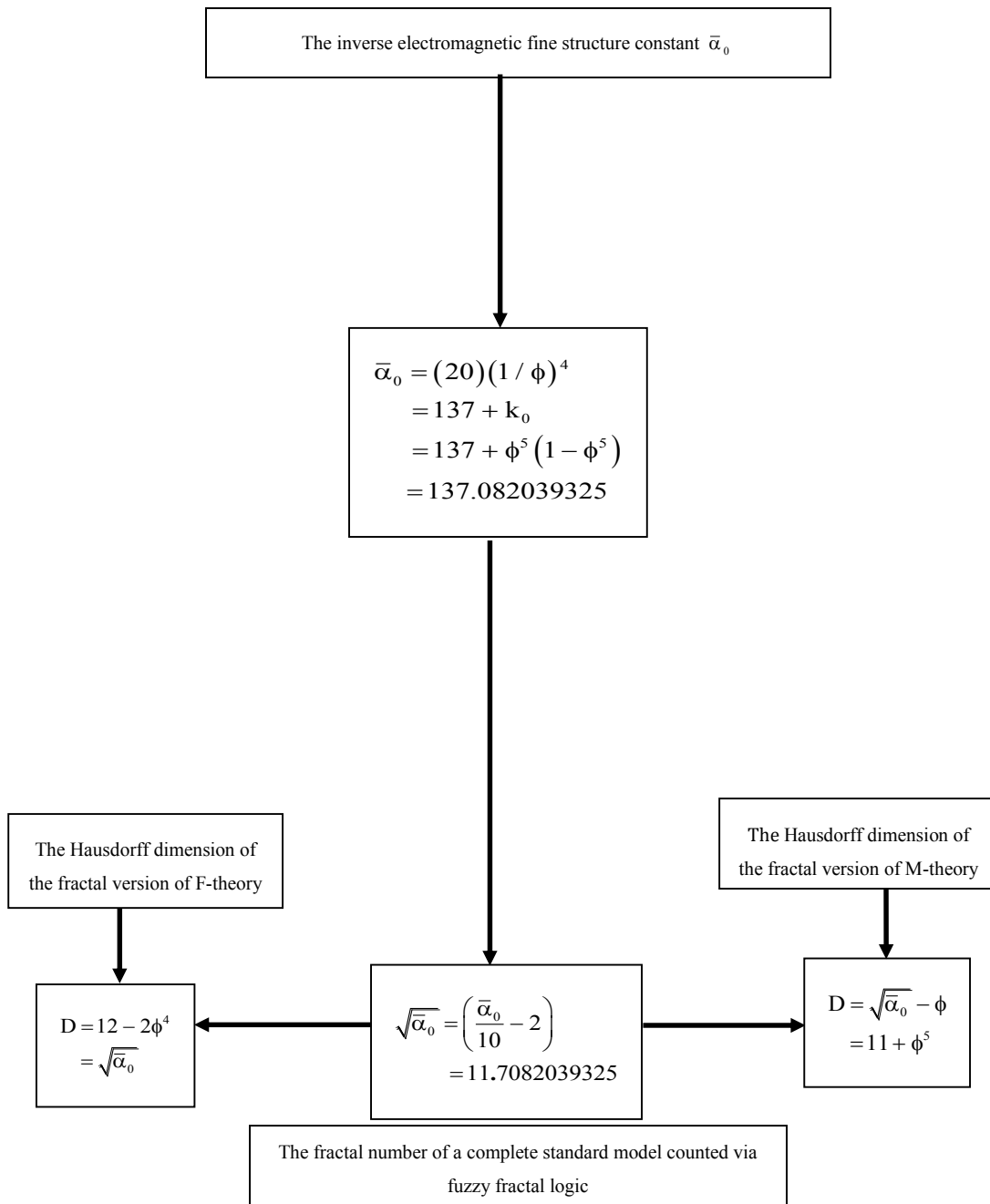


CHART (2)



Electromagnetic interpretation of the fractal version of Witten's M-theory and Vafa's F-theory.

CHART (3)

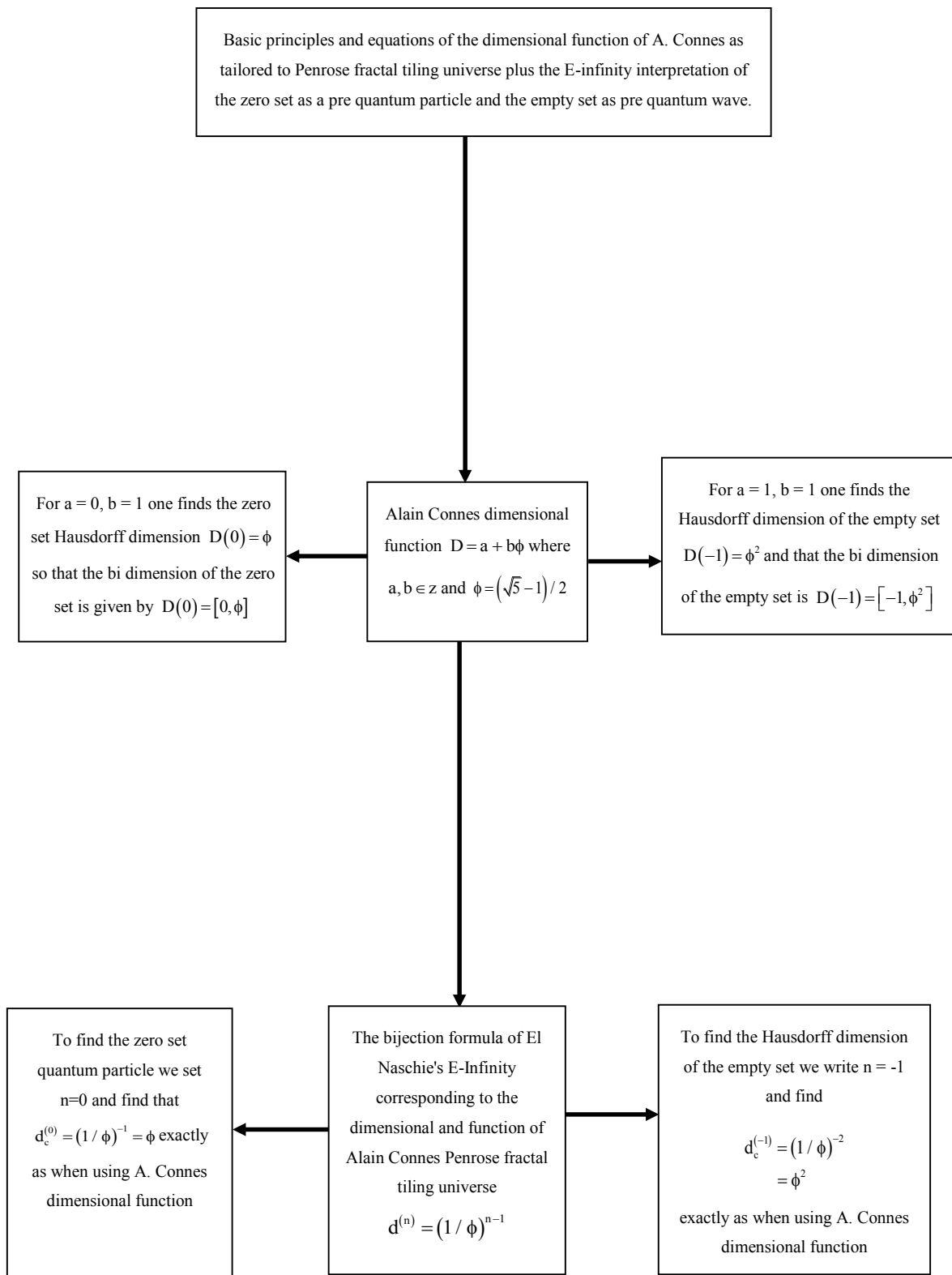


Table 1. The fractal - Cantorian spacetime Hausdorff dimensions of the fractal version of C. Vafa's F-Theory discovered recently within the framework of E-Infinity Cantorian theory compared with the other scale invariant theories of Einstein, Kaluza-Klein and Witten.

The original "integer" theory	The corresponding scale invariant fractal - Cantorian theory	Interpretation of the Theory	Continued fraction representation of the Hausdorff dimension of the theory
Einstein's spacetime D = 4	$D = 4 + \phi^3$ where $\phi = (\sqrt{5} - 1) / 2$	Scale invariant fractal spacetime	$4 + \phi^3 = 4 + \frac{1}{4 + \frac{1}{4 + \dots}}$
Kaluza-Kein spacetime $D = 4 + \phi + \phi^2 = 5$ where $\phi = (\sqrt{5} - 1) / 2$	$D = 5 + \phi^3$	Scale invariant fractal spacetime	$5 + \phi^3 = 5 + \frac{1}{4 + \frac{1}{4 + \dots}}$
Witten's M-Theory D=11	$D = 11 + \phi^5$ where ϕ^5 is Hardy's quantum entanglement	Scale invariant correlated spacetime	$11 + \phi^5 = 11 + \frac{1}{11 + \frac{1}{11 + \dots}}$
C. Vafa's F-Theory D = 12	$D = 12 - 4\phi^4 = \sqrt{\bar{\alpha}_0} = 11.7082039325$ where ϕ^4 is the Unruh topological temperature and $\bar{\alpha}_0$ is inverse electromagnetic fine structure constant $\bar{\alpha}_0 = (20)(1 / \phi)^4 = 137 + k_0$ and $k_0 = \phi^5 (1 - \phi^5)$.	This is the most remarkable case connected to $\bar{\alpha}_0$ as well as Schoenberg's 12 tunes music and the standard model	$11 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \dots}}} = 11.7082039325$

Table 2. Comparison between the self-similar i.e. scale invariant fractal versions of Einstein, Kaluza-Klein, witten, Heterotic superstrings, 'tHooft-Veltman-Wilson and El Naschie spacetime.

Name of spacetime theory	Smooth spacetime dimension	Non-smooth (fractal) spacetime dimension of El Naschie
Einstein	D = 4	$D = 4 + \phi^3 = 4 + \frac{1}{4 + \frac{1}{4 + \dots}}$ $\phi = (\sqrt{5} - 1) / 2 = 0.618033989$ <p>Note that the above table is based on A. Connes dimensional function $D = a + b\phi$ where $a, b \in \mathbb{Z}$ and ϕ is the golden mean. In this way we have the following bi dimensions $D(0) = (0, \phi)$ and $D(-1) = (-1, \phi^2)$ for the zero set quantum particle and the empty set quantum wave respectively table (-)</p>
Kaluza-Klein	D = 4 + 1 = 5	$D = 4 + \phi^3 = 5 + \phi^3 = 5 + \frac{1}{4 + \frac{1}{4 + \dots}}$

Witten M-theory and super gravity	D = 11	$D = 11 + \phi^5$ $= 11 + \frac{1}{11 + \frac{1}{11 + \dots}}$
D. Gross et al Heterotic superstring theory	D = 26, 16, 10	$D = 26 + k, k, 10$ $k = \phi^3 (1 - \phi^3) = 0.18033989$
'tHooft-Veltman-Wilson dimensional regularization spacetime	D = 4 - ε, ε ≪	D = 4-k

Table 3. The mass-energy equivalence equations according to the theories of Nikolay Umov, Friedrich Hasenohrl, Einstein-Lorentz, Poincare and E-Infinity of El Naschie.

Theory	Equation	Comments
Nikolay Umov mass-energy Aether theory 1873	$E = kmc^2$ $0.5 \leq k \leq 1$	Formally when $k = 1/2$ and $c = v$ we have Newton's kinetic energy. For $k = 1$ we have Einstein's maximal energy density. However "ordinary" energy density and dark matter energy density requires $k < 1/2$
Friedrich Hasenohrl Electromagnetic apparent mass 1904	$E = \frac{3}{4} m_0 c^2$	Since $3/4 = 0.75$ this is very close to the pure dark energy density found by El Naschie, namely 0.733111.
Einstein-Lorentz and Poincare maximal energy 1905	$E = mc^2$	
El Naschie ordinary cosmic energy density 2012	$E(0) = (\phi^2 / 2)(mc^2)$ $= mc^2 / 22.1803398$ $= mc^2 / 22$	Note that $(\phi^2 / 2) + (5\phi^2 / 2) = 1$ which means this result, together with that of dark energy, constitutes a proof of the correctness of $E = mc^2$ of Einstein.
El Naschie cosmic dark energy density 2013	$E(D) = (5\phi^2 / 2)(mc^2)$ $= (mc^2)(21 + k) / (22 + k)$ $= mc^2 (21 / 22)$	Note that E(D) consists again of two parts, namely dark matter energy density 0.2218 and pure dark energy density 0.73311 both found by El Naschie.