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Maxwell's Equations, Emotions and Beta Wave

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***Abstract* – This work studies the solutions of Maxwell's equations in the case of the presence of a charge and a current. Those equations model the propagation and interaction of electromagnetic fields. As an application, we present the idea of decoding emotions in brain waves. Indeed, emotions are an electrochemical activity of the limbic brain that automatically generates an electromagnetic wave.**

***Keywords* – Mathematical Modeling, Emotions, Limbic System, Maxwell's Equations, Beta Wave.**

I. MAXWELL'S EQUATIONS

Maxwell's equations constitute the basic postulates of electromagnetism, with the expression of the Lorentz electromagnetic force. These equations are in the numbers of 4. And they are respectively: the Maxwell-Gauss equation, the Maxwell-Faraday equation, the Maxwell-Flux equation, the Maxwell-Ampere equation. These equations show in particular that in stationary conditions, the electric and magnetic fields are independent of each other, while they are not in variable conditions. In the most general case, we must therefore speak of the electromagnetic field. Maxwell succeeds in writing them in the form of integral equations.

In the article [3], Y. Annabi explains the mathematical model. It is composed of a system of equations of linear operators as described in the book [1]. Neuronal brain sources work like a dipole. This allows the EEG and MEG sources to be set in a three-dimensional coordinate system. Assuming that the medium is homogeneous and isotropic, the researchers obtain simple formulas of electro-magnetic behavior of the brain. In this conducting medium, isolated in the air, the electric field E and the magnetic field B verify Maxwell equations. Studies done on electrophysiology, especially in EEG, use the quasi-static model because of the low signal frequency and the low electrical capacity of the tissues of the head. In fact, the frequency of the EEG signals varies most often between 0.5 Hz and 100 Hz. On the other hand, the tissues of the head appear as passive conductors so researchers consider that the electric currents and magnetic fields behave at any moment stationary. The cerebral electro-magnetic fields (E, B) verify the following Maxwell equations :

$$\begin{cases} \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla . E &= \frac{\rho}{\varepsilon_0} \\ \nabla \times B &= \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \\ \nabla . B &= 0 \end{cases} \quad (1)$$

With

1. J is the current density.
2. ρ is the charge density.
3. ε_0 is the permittivity of the vaccum. $\varepsilon_0 = 8.85\ 10^{-12}\ Fm^{-1}$
4. μ_0 is the permeability of the vaccum. $\mu_0 = 4\pi\ 10^{-7}\ Hm^{-1}$

The first equation, known as the Maxwell-Faraday equation, gives the relation between the circulation of the electric field along the solid circuit and the temporal change magnetic field flux through a surface that rests on this circuit. The second equation, known as the Maxwell-Gauss equation, expresses the fact that the flow of an electric field through a closed surface is related to the electric charge contained inside this surface. The third equation, known as the Maxwell-Ampère equation, expresses the relation between the circulation of the magnetic field along the solid circuit and the leakage current through a surface resting on this circuit. Finally, the fourth equation, called the Maxwell-Flow equation, expresses that the magnetic field flow through any closed surface is zero.

The work is therefore, reduced to solving Maxwell's equations in conditions similar to that of vacuum but with the presence of charge ρ and current$J$.

II. Solution

The solutions of Maxwell's equations written in this paragraph are extracted from [4]. In the same document, there is a more precise study concerning the calculation details of the solutions in different cases. In the following, we present a resolution, with the hypothesis of the existence of a spherical symmetry. Therefore, we will use the spherical coordinates (r, θ, φ). In addition, it will be assumed that the variations are related only to the coordinate r.

2.1 *From Maxwell's System to the D'alembert Equation*

We have,

$$\nabla \times (\nabla \times E) = \nabla.(\nabla. E) - \Delta E \quad (2)$$

According to the Maxwell-Faraday Equation, we have

$$\nabla \times \left(-\frac{\partial B}{\partial t}\right) = \nabla.(\nabla. E) - \Delta E \quad (3)$$

Using the Maxwell-Gauss Equation, we obtain

$$-\frac{\partial}{\partial t}(\nabla \times B) = \nabla.\left(\frac{\rho}{\varepsilon_0}\right) - \Delta E \quad (4)$$

Then, the Maxwell-Ampère equation applied to the first member of the equality makes it possible to write

$$-\frac{\partial}{\partial t}\left(\mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right) = -\Delta E \quad (5)$$

In other words,

$$-\mu_0 \frac{\partial J}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = -\Delta E \quad (6)$$

Then, the Maxwell-Ampère equation applied to the first member of the equality makes it possible to write.

Taking into consideration the hypotheses put forward at the beginning, the problem of identifying the electric field E reduces to solving the following d'Alembert equation:

$$\Delta E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 J \quad (7)$$

A similar reasoning is applied to the magnetic field B. The problem therefore comes down to solving the equation:

$$\Delta B - \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} = -\mu_0 J \quad (8)$$

Conclusion 1:

All the calculations lead to the solution of the d'Alembert equation in the form,

$$\Delta . F - \frac{1}{c^2} . \frac{\partial^2 F}{\partial t^2} = \pm \mu_0 J \quad (9)$$

with $\mu_0 \varepsilon_0 = \frac{1}{c^2}$ and F being able to represent the electric fields E or the magnetic fields B.

2.2. *A solution thanks to vector and scalar potentials*

We will solve the equation (9) using the vector potential A and scalar potential U defined by :

$$\begin{cases} B & = & \nabla . A \\ E & = & -\nabla . U - \frac{\partial A}{\partial t} \end{cases} \quad (10)$$

Thanks to Maxwell's equations (1) and the definition (10), these two potentials verify the following equations :

$$\begin{cases} \Delta A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} & = & -\mu_0 J \\ \Delta U - \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} & = & -\frac{\rho}{\varepsilon_0} \end{cases} \quad (11)$$

In a spherical reference frame, the solutions are of the form :

$$\begin{cases} A(r,t) & = & \frac{\mu_0}{4\pi} \iiint_\Omega \frac{J(t-r/c)}{r} \, dv \\ U(r,t) & = & \frac{1}{4\pi\varepsilon_0} \iiint_\Omega \frac{\rho(t-r/c)}{r} \, dv \end{cases} \quad (12)$$

2.3. *A Solution in Harmonic Regime*

Maxwell's equations are not always solvable theoretically. In this paragraph, we present a solution calculation, adding two hypotheses.

1. The electromagnetic field is generated by sinusoidal currents.
2. All quantities have a time dependence expressed by $e^{i\omega t}$.

Thanks to these assumptions, the equation (9) becomes :

$$(\Delta + k^2) . F = \pm \mu_0 J \quad (13)$$

with $= \frac{\omega}{c}$. After calculations carried out thanks to the Green function of the operator $(\Delta + k^2)$, the expression of the vector potential is

$$A = \frac{\mu_0}{4\pi} \iiint_\Omega \big(J(M_0) . \Psi(r)\big) \, dv \quad (14)$$

With

$$\Psi(r) = \frac{e^{-ikr}}{r} \quad (15)$$

The magnetic fields B and the electric fields E are expressed by the formulas :

$$\begin{cases} B = \nabla \times A \\ E = \frac{1}{4\pi j\omega\varepsilon_0}(\nabla.(\nabla.A) + K^2.A) \end{cases} \quad (16)$$

III. Emotions and Beta wave

Let's apply the modeling approach explained in the book [2] and let's observe the phenomenon. Brain waves are generated by the activity of the nervous system of the brain. They can be recorded by machines such as electroencephalograms and magneto-encephalograms. In the article [3] , Y. Annabi explains the applications of EEG and MEG machines in different fields such as cognitive sciences, psychology, neuroscience and neuromarketing. These machines make it possible to visualize brain waves in the form of plots. There are six types of brain waves: alpha, beta, gamma, delta and theta waves. The classification is carried out according to the frequency of these waves. They are measured in hertz (Hz). One hertz equal to one ripple per second. The frequencies of delta waves vary between 0.5 to 4 Hz. They are generated during deep sleep, without dreams. The frequencies of theta waves vary from 4 to 7 Hz. Those are the consequence of a deep relaxation, in full awakening, achieved especially by experienced meditators. The frequencies of alpha waves evolve from 8 to 13 Hz. These are the result of light relaxation and calm awakening. The frequencies of beta waves vary from 14 to 30 Hz. They usually appear when the nervous system is active and are therefore associated with sensory stimulation and mental activity. They are present especially during periods of intense activity, concentration or anxiety. Finally, the gamma waves have a frequency between 30 and 100 Hz, they are rare. They testify to a great brain activity, as during creative processes or problem solving. They are different from gamma rays, emitted by the nucleus of atoms. These waves are classified in the figure 1.

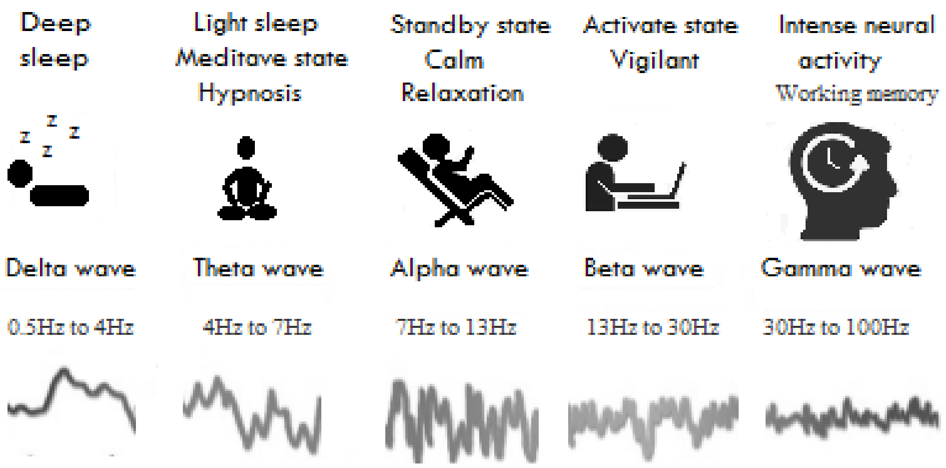

Fig. 1. Brain waves.

Emotions are an electro-chemical signals that originates in the brain and drives reactions in the body. More precisely, emotions are generated in the limbic system, as explained in the figure 2. Emotions exist in most brainwaves; however, there is a hypothesis that they can be decrypted in beta waves, because it is a state of conscious activity. The limbic system, is also called limbic brain or emotional brain. It represents an anatomical and functional interface between cognitive life and vegetative life. The main components of the limbic system are the following cerebral cortex and subcortical structures: the hippocampus, the amygdala, the cingulate convolution (or gyrus), the fornix, the hypothalamus. The limbic system has mental functions other than emotions, such as memory.

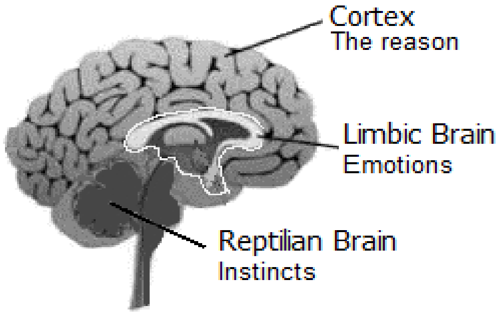

Fig. 2. Limbic brain.

IV. Conclusion

This article presents emotions as brain waves by hypothesizing that they can be deciphered in beta waves. Brain waves are modeled mathematically by Maxwell's equations. A solution is drafted.

References

[1] Annabi, Y. Introduction to linear operators and nonlinear operators, Book, European University Editions, 2016, ISBN : 978-3-639-50779-9.
[2] Annabi, Y. Introduction to applied mathematics, European University Editions, 2012, ISBN : 978-3-8417-8154-3.
[3] Annabi, Y. Multidisciplinary researches using the magneto-electro-encephalogrphy, International Journal of Emerging Technology and Advanced Engineering, Volume 9, Issue 7, 2019.
[4] Vaudon, P. Solution of Maxwell's equations, Research master's course in high-frequency and optical telecommunications, University of Limoge.

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